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# RELATIVISTIC ORBIT DETERMINATION WITH THE RMI (RELATIVISTIC MOTION INTEGRATOR) SOFTWARE FOR THE LISA MISSION

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**Abstract.** The LISA constellation aims at the detection of gravitational waves. Precise ephemerides of its spacecraft are needed for orbit determination and for the so-called Time Delay Interferometry (TDI) data-pre-processing. We use the Relativistic Motion Integrator (RMI) to compute LISA ephemeris and confront this relativistic approach with a classical orbit model.

## 1 Introduction

The LISA mission (LISA, 1998), to be launched in 2015, is an interferometer that aims at the detection of gravitational waves in the $[10^{-4}, 10^{-1}]$ Hz frequency band. LISA is formed by three spacecraft, interconnected by double laser links. Precise ephemerides of LISA spacecraft are needed not only for the sake of orbit determination but also to compute the photon flight time in laser links between spacecraft, required in data TDI pre-processing (Nayak & Vinet 2005; and references therein) which aims at lowering laser frequency and optical bench noises down to LISA specifications in order to reach the gravitational wave-detection level.

Before the present work, only *classical* ephemerides for LISA were available. Hence, relativistic effects in LISA orbit determination needed to be quantified.

## 2 Orbit models

The characteristics of the spacecraft orbits are the following: drag-free motion at an average interdistance of $L = 5$ million kilometers, rotation of the triangular constellation around its center of mass and around the Sun. We considered some simplifying assumptions, namely: no non-gravitational forces are applied on the spacecrafts, each one perfectly follows a free-falling test mass, itself perfectly shielded; planetary perturbations are neglected and the Sun is assumed non-rotating and spherical.

The generic method of the *Relativistic Motion Integrator (RMI)* (Pireaux et al. 2006) is to numerically integrate, instead of Newtonian equations plus relativistic corrections, the exact relativistic equation of motion (for a given metric, corresponding to a gravitational field at first Post-Newtonian -PN- order or higher). The Christoffel symbols, present in the relativistic equations, contain all gravitational effects, classical and relativistic, at corresponding order of the selected metric. According to the above cited LISA simplified assumptions, there is no non-gravitational force (geodesic motion), and we use the BCRS (Barycentric Coordinate Reference System) coordinates and Post-Newtonian (PN) metric IAU2000, as recommended by the IAU (International Astronomical Union).

Our generic *classical model* numerically integrates Newton‘s second law of motion, *without* any PN relativistic corrections, around the central body.

In particular for the LISA simplified assumptions, it implies Keplerian motion around the Sun without planets, with a LISA plane angle of $\pi/3 + 5/8 \cdot 1/60$ to minimise armlength variations, common orbit parameters for the three spacecraft (semi-major axis of 1 A.U., small orbital eccentricity, orbit inclination and orbital period of about 1 year), with a $2\pi$ phase off-set, as decribed in reference (Nayak et al. 2006).

Initial conditions for LISA spacecraft in RMI computation are the same as the selected classical Keplerian initial positions and velocities.

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## 3 Comparison between numerical relativistic and classical orbit models for LISA

With the orbit models described in Section 2, we show that the numerical classical model for LISA orbits in the gravitational field of a non-rotating spherical Sun without planets can be wrong, with respect to the numerical relativistic version of the same model, by as much as about ten kilometers in radial distance during a year and up to about 60 kilometer in along track distance after a year (Fig. 1 Left and Center). Relativistic effects in spacecraft inter-distance are relevant since they lead to a significant correction in the photon time transfer $t_{ij}$ between two spacecraft $i$ and $j$, which is used in TDI (Fig. 1 Right). This correction adds up to (and is several orders of magnitude higher than) the relativistic effects in laser links for a classical LISA orbitography, studied in reference (Chauvineau et al. 2005). The above numerical results obtained with RMI were confirmed with an analytical 1PN development at first order in the small LISA eccentricity, $e_{LISA} \simeq 0.0096$, (Pireaux & Chauvineau 2008).

## 4 Conclusions and perspectives

LISA is a very complex mission and the TDI method must be validated. Hence the need for a LISA simulator. Such is LISACode (Petiteau et al. 2008); but the orbit model presently implemented in LISACode is classical, while the laser link model is relativistic. In the present paper, using RMI, we have quantified and demonstrated the relevance of the relativistic effects in LISA orbit determination.

LISA is a relevant example to use RMI. The strength of the RMI method is that it can be used to compute relativistic orbits for different missions (whether barycentric or planetocentric); only the central body parameters and initial conditions, mission parameters (number of satellites or planets) in the corresponding RMI modules would change. If the IAU 2000 metric is updated, only the metric module in RMI needs to be updated, no additional analytical developments must be recomputed. Indeed, RMI includes any gravitational contribution at the corresponding order of the metric (whether 1PN or higher). RMI is a coherent native relativistic approach, and it should be preferred to "Newton + relativistic correction" methods. In the future, the RMI software will use a symplectic integrator, instead of presently used Runge-Kutta of order 8. Non-gravitational forces will be implemented, as well as planetary perturbations, in particular for the LISA mission.

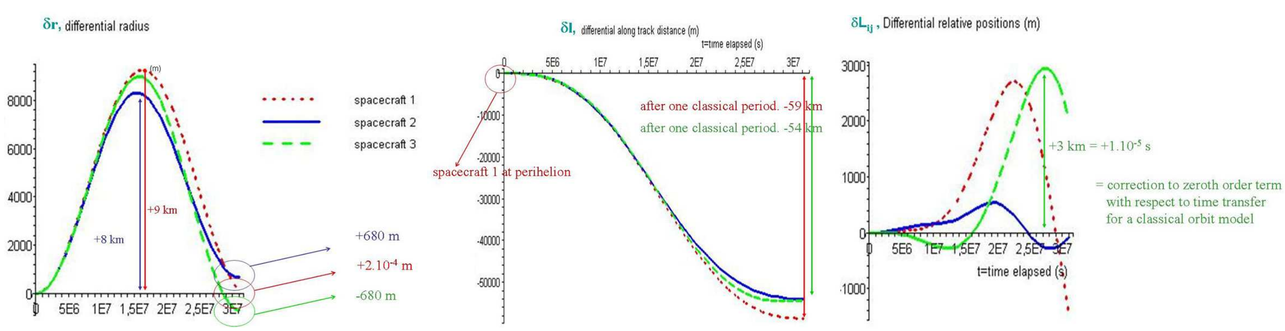


**Fig. 1.** Difference between numerical relativistic and classical position ephemerides for the LISA mission. **Left** : In radial barycentric distance ($\delta r$). **Center**: In along track distance ($\delta l$). **Right**: In the relative distance between spacecraft, $\delta L_{jk} = \delta(r_k - r_j)$ with $j, k = 1, 2, 3$, $j \neq k$.

## References

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