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NUMERICAL STUDIES OF THE VISHNIAC INSTABILITY IN SUPERNOVA REMNANTS

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Abstract. Vishniac instability has been theoretically studied in supernova remnants where it is supposed to explain the fragmentation of the interstellar medium. However its role is not fully demonstrated in these objects. Numerical simulations with the HYDRO-MUSCL hydrodynamic code has been realised to simulate this instability in order to compare the numerical growth rate with the Vishniac analytical solution.

1 Introduction

Among the instabilities arising in astrophysical systems, and in particular in supernova remnants (SNRs), the Vishniac instability is not very well known. In their original analysis of global perturbations, Vishniac (1983) and Ryu & Vishniac (1987) identified the criteria that allows to compute the growth of a perturbation in a thin shell of shocked matter. The instability depends on the direction of two opposite forces: the thermal pressure p_{th} due to hot SNR gas (pushe outwards) and the ram pressure p_{ram} due to the accretion of the surrounding interstellar matter (ISM) on the shock front (compresse inwards). If the ISM is uniform, the two pressures keep the same directions and the shock front is stable; but in the case of non-uniform ISM, the shock front is distorted, the two forces do not counterbalance and oscillations can develop and grow. In a linear stability analysis, Vishniac (1983) used the infinitely thin shell approximation to study the perturbation equations and numerically solved the corresponding system. In a recent study (Cavet *et al*, 2007) an analytical solution for the growth rate has been determined in this case, but this approximation does not allow to access the general instability criteria. Aiming to investigate the more realistic physical cases of a thin shell with finite thickness, the growth rate is calculated through numerical simulations of perturbed radiative blast waves, and compared with the analytical one.

2 Numerical simulations

Numerical simulations are performed with the HYDRO-MUSCL code developed by our team. This Eulerian code solving hydrodynamic equations, uses a regular cartesian grid and an adaptative time step. The underlying numerical method is a MUSCL-Hancock finite volume scheme and a HLLC Riemann solver (Toro, 1999). In order to perform 2D cylindrical hydrodynamic simulations for the propagation of a shock wave in the ISM, we induce an initial explosion by depositing a strong amount of energy in the form of p_{th} at the center of a box. In a first simulation, we study the initial phase of the shock evolution where the gas is adiabatic (the polytropic index is $\gamma = 5/3$) and then the shock radius can be approximated with the Sedov law (see Keilty *et al*, 2000). The simulation is stopped at (t_0, r_0) *i.e* when the radius evolves according to the self-similar solution and when the density on the shock front reaches the strong shock limit. We use this result as input data in the following simulation. In the second calculation, axial velocities are directly modified in data in order to obtain the snowplow radius evolution and radiative losses are taken into account by $\gamma = 1.1$. We introduce high-density spots ($\rho_{spot} \propto \rho_{shock}$) ahead of the shock wave to perturb the shock front and let the system evolve. The analytical form of this perturbative spots are (Ryu & Vishniac, 1987): $\widetilde{\rho} = \rho/\rho_{ISM} = \delta\tilde{\rho}_i(\xi)\ Y_{lm}(\theta, \Phi)\ t^s$.

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To determine the initial profile of these spots, we have to evaluate the function $\delta\tilde{\rho}_i(\xi)$ of the dimensionless spatial parameter ξ. In the analytical part, this function is not explicitly estimated and only determined by numerical means. We have obtained an approximated value $\delta\widetilde{\rho}_i(\xi) \approx -(2a-2)\xi^{2a-3}$ with the method developed in Kushnir *et al* (2005). Then in the numerical simulations we introduce a finite number of spots to take into account the perturbative mode number l and we add a bi-dimensional spot shape in order to reproduce the deformation profile according to r, θ, ϕ.
Numerically we observe that the two spots create radial oscillations both on density and pressure, and density perturbation on the shock front. We remarke also that in the two local pressure deformations of the shell shape, p_{th} and p_{ram} are not aligned as in the analytical pattern. Thus in this numerical configuration, the Vishniac instability criteria γ and $\delta\widetilde{\rho}_i(\xi)\, Y_{lm}$ are fulfilled. In order to observe the evolution of the growth rate s of this small perturbations and to compare s with the theoretical results, we let run the simulation during $t = 10 \times t_0$. In Fig. 1 we superpose ten snapshots of the density profile projected on the y-axis and normalized for the shock

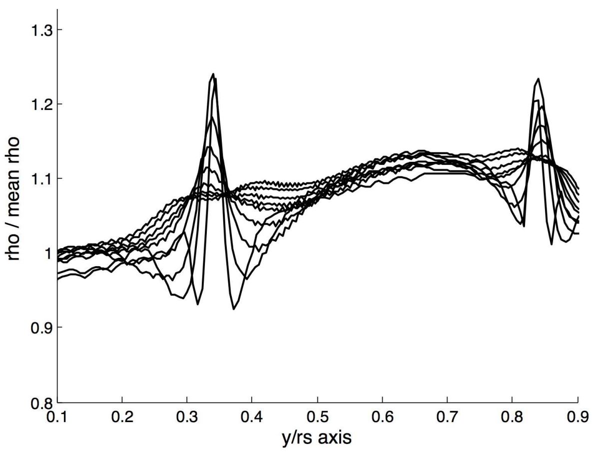

Fig. 1. Ten dimensionless density profiles $\widetilde{\rho}$ versus y-axis

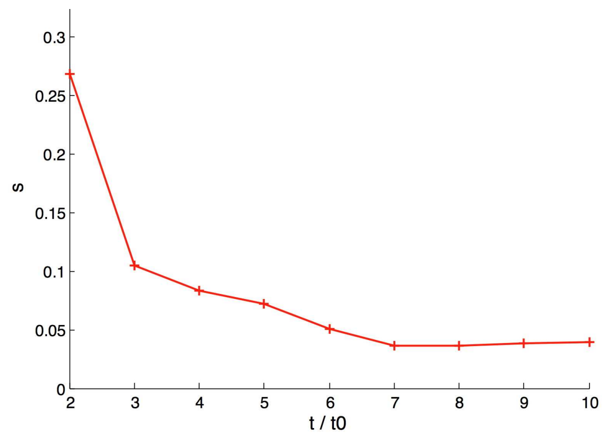

Fig. 2. Evolution of perturbation growth rate s versus t

front value which enables to measure the normalized peak oscillations $\delta\widetilde{\rho}$ during this period. We estimate the growth rate by the relation: $s = [\ln(\delta\widetilde{\rho}) - \ln(\delta\tilde{\rho}(1)\ Y_{lm})]\,/\ln t$ where $\delta\tilde{\rho}(1) = 1.8$. Figure 2 shows the evolution of the growth rate $s(t)$ and its stabilization to a limit value $s = 3.7 \times 10^{-2}$ clearly observed. Compared to Vishniac prediction, this value seems too low. However, we have chosen a small mode number $l \sim 8$ due to the presence of only two high density spots. If we increase the number of initial spots, l is also increasing and can reach the optimal mode number $l = 40$ of the theoretical pattern. However due to numerical constraints it is not easy to multiply initial high-density spots on the way of the thin shell. In future simulations, we will improve the stability study using a multi-processor version of HYDRO-MUSCL code. We will combine this new simulations with profiles of perturbed shell designed by analytical tools that will enable a better control of spatial shape and thus of the value of the mode number l. Furthermore an experiment of the Vishniac instability realized by our team is planned on the LIL high-power facility (Bordeaux, France) in 2009.

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