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IMPACT OF LARGE-SCALE MAGNETIC FIELDS ON STELLAR STRUCTURE AND PROSPECTIVES ON STELLAR EVOLUTION

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Abstract. The influence of large-scale magnetic fields on stellar structure and stellar evolution is semi-analytically considered. The magnetic field is derived for a given axisymmetric azimuthal current, and is non force-free, acting thus directly on the stellar structure by modifying the hydrostatic balance. We discuss the relative importance of the various terms associated with the magnetic field in the mechanical and thermal balances before implementing its effects in a 1D stellar evolution code in a way that preserves its geometrical properties. Our purpose is illustrated by the case of an internal magnetic field matching at the surface of an Ap star with an external potential and multipolar magnetic field.

1 Introduction

Though the traditional picture describing the stellar structure and the stellar evolution has succeeded in answering to many questions of the stellar physicists during the last century, some facts indicate today that there is a need for a model that goes far beyond the standard stellar evolution model. The discrepancy between the helioseismology-deduced sound speed and the one found using new solar abundances (Turck-Chièze *et al.*, 2004), or the flat rotation profile observed through helioseismic inversions in the radiation zone (Mathur *et al.* 2008) are two major examples highlighting the necessity to introduce the influence of the rotation, the magnetic field and the internal waves in the equations describing the stellar structure, to reach a complete physical picture of stellar interiors.
We here focus on the impact of a large-scale magnetic field on the stellar structure and we present how the geometry of the magnetic field can be modeled by considering non force-free magneto-hydrostatic (MHS) equilibria using a Grad-Shafranov approach (see also our PNST contribution). This allows us to discuss the relative importance of the various terms associated with the magnetic field. A pertubative treatment is performed on an Ap-type star with an axisymmetric dipolar magnetic field matching at the stellar surface with a potential field. The perturbations of the structural quantities are obtained and the limits of this approach underlined.

2 Non force-free magneto-hydrostatic equilibria

The more natural way to take into account the dynamical processes that might influence the stellar evolution secularly, namely the influence of the rotation and the magnetic field, in a unidimensional stellar evolution code, is to project their respective terms acting upon the structure on the vectorial spherical harmonics basis (Mathis & Zahn 2004, 2005). In the case of the magnetic field however, the poloidal and toroidal components remain arbitrary in the model when initial conditions are considered. Assuming magneto-hydrostatic equilibria provides constraints on these magnetic initial configuration.
Moreover, since observations reveal that about 5 percents of the A-type stars present an external magnetic field organized over large scales, so a magnetic field of dynamo or fossil origin lies certainly in a non-negligible proportion of main sequence star's radiation zones and can probably influence significantly their evolutionary tracks, since they are likely to be non force-free. We provide here a way to model any MHS equilibrium that fulfill all the exposed requirements, by supposing any prescription for the toroidal current.

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2.1 Axisymmetric magnetic field and Grad-Shafranov-Poisson equation

In the present work, we assume that the magnetic field ($\boldsymbol{B}$) is axisymmetric. In this case we can express its components in function of a poloidal magnetic flux fonction $\Psi(r,\theta)$ and of a toroidal potential function $F(r,\theta)$ ($\hat{\mathbf{e}}_\varphi$ being the azimuthal unit vector) such that $\boldsymbol{B}(r,\theta)$ remains divergenceless :

$$\boldsymbol{B} = \frac{1}{r\sin\theta}\nabla\Psi\times\hat{\mathbf{e}}_\varphi + \frac{1}{r\sin\theta}\,F\,\hat{\mathbf{e}}_\varphi. \tag{2.1}$$

On the other hand, Ampere's law in the MHD classical approximation is given by $\nabla\times\boldsymbol{B} = \mu_0\,\boldsymbol{j}$, where $\boldsymbol{j}$ is the current density and μ_0 the vacuum permeability. Projected along the azimuthal direction, one gets

$$\mu_0\,j_\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta\right) - \frac{1}{r}\frac{\partial B_r}{\partial\theta}. \tag{2.2}$$

It leads to the Grad-Shafranov-Poisson equation (hereafter called the GSP equation):

$$\Delta^*\Psi = -\mu_0 r\sin\theta\,j_\varphi \quad \text{where} \quad \Delta^*\Psi \equiv \frac{\partial^2\Psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\theta}\right). \tag{2.3}$$

2.2 The current density function

The formalism is derived in order to take as an input of the model any given toroidal current density $j_\varphi(r,\theta)$. Here, to illustrate our purpose a simple function is chosen, taken on the form: $j_\varphi(r,\theta) = j_{\varphi_0}\,j_{\varphi_r}(r)\,j_{\varphi_\theta}(\theta)$. The radial function is taken as $j_{\varphi_r} = \sin\left(\pi r/R_*\right)/\left(\pi r/R_*\right)$ if $0 \le r \le R_*$ (R_* is the star's radius) and $j_{\varphi_r} = 0$ otherwise and the angular function as dipolar: $j_{\varphi_\theta} = \sin\theta$. The magnetic strength B_0 giving the amplitude j_{φ_0} is determined according to the fact that at its maximum, the magnetic pressure is equal to $B_0^2/2\mu_0$.

2.3 Non Force-Free Condition

It is supposed that magnetic fields force-free everywhere, though they are stable, are unlikely to exist in stellar interiors, since such fields require an unrealistic Lorentz force at the stellar surface in order to avoid the field itself to vanish. Let us start then from the non force-free MHS equilibrium :

$$\rho\,\boldsymbol{g} - \nabla P_{\text{gas}} + \boldsymbol{F_{\mathcal{L}}} = \boldsymbol{0}, \tag{2.4}$$

where ρ is the density, $\boldsymbol{g}$ the local gravity field, P_{gas} the gas pressure and $\boldsymbol{F_{\mathcal{L}}} = \boldsymbol{j}\times\boldsymbol{B}$ the Lorentz force.
Requiring the toroidal component of the Lorentz force $F_{\mathcal{L}_\varphi}$ to vanish everywhere writes as $\frac{\partial\Psi}{\partial r}\frac{\partial F}{\partial\theta} - \frac{\partial\Psi}{\partial\theta}\frac{\partial F}{\partial r} = 0$. The non-trivial values for F are thus obtained by setting $F(r,\theta) = F(\Psi)$. For a regular function, we can make a serial expansion $F(\Psi) = \sum_{n=0}^{\infty}\alpha_n\Psi^n$. We only keep the first-order term since the zeroth-order one leads to a singular toroidal magnetic field at the center. Let then consider $F(\Psi) = \alpha\Psi$ where α is taken as $\alpha = 1/R_*$ based on a dimensional analysis. The magnetic field topology is now completely determined by the function Ψ.

2.4 The GSP solutions

We solve the equation (2.3) using the Green's functions method (cf. Morse & Feshbach, 1953; Payne & Melatos, 2008, Duez *et al.*, 2008b). The general expression for the flux function Ψ inside the star is given by

$$\Psi(r,\theta) = \sum_{l=0}^{\infty} -\mu_0\,\mathcal{N}_l^{-1}\sin^2\theta\,C_l^{3/2}(\cos\theta)\int_0^{R_*} g_l^*(r',r)\left[\int_0^{\pi} j_\varphi(r',\theta')\,C_l^{3/2}(\cos\theta')\sin^3\theta'd\theta'\right]r'^3dr', \tag{2.5}$$

where $C_l^{3/2}(\cos\theta)$ is the Gegenbauer polynomial of latitudinal order l and the normalization coefficient is defined by $\mathcal{N}_l = \frac{2(l+1)(l+2)}{(2l+3)}$. For multipolar boundary conditions the Green's function is given by $g_l^*(r,r') = -\frac{1}{(2l+3)}\frac{r^l}{r'^{l+1}}$ if $r < r'$, $g_l^*(r,r') = -\frac{1}{(2l+3)}\frac{r'^{l+2}}{r^{l+3}}$ if $r > r'$.
Outside the star, the expression obtained for the flux function is:

$$\Psi_{\text{ext}}(r,\theta) = \sum_{l=1}^{\infty}\frac{\alpha_l}{l}\sqrt{\frac{2l+1}{4\pi}}\frac{R_*^{l+1}}{r^l}\sin\theta\,P_l^1(\cos\theta), \tag{2.6}$$

$P_l^1(\cos\theta)$ being the associated Legendre polynomial of azimuthal degree $m = 1$. The coefficients α_l are determined so that the internal solution $\Psi_{\text{int}}(r,\theta)$ matches the potential external one $\Psi_{\text{ext}}(r,\theta)$ at $r = R_*$.

3 Hierarchy of the physical quantities aimed to be implemented in a stellar evolution code

3.1 The necessity to take into account the geometrical nature of the field

3.1.1 Goldreich's β parameter

This parameter, defined by Goldreich (1991) as

$$\beta_{\mathrm{Gold.}} = \left(< B_{\mathrm{h}}^2 > - < B_{\mathrm{r}}^2 >\right) / \left(< B_{\mathrm{h}}^2 > + < B_{\mathrm{r}}^2 >\right) \tag{3.1}$$

where the r and h subscripts stand for the vertical and horizontal directions, has been understood by several authors as a parameter aimed to mimic the geometrical effects of the field once implemented in 1D models. Computing this quantity for a given axisymmetric MHS equilibrium (see Fig. 1, left), we show that this quantity has at least to be taken as a function of the radius to reproduce correctly the field's geometry properties.

3.1.2 Magnetic pressure gradient *versus* magnetic tension force

We would like to pinpoint here also the necessity to take into account the whole Lorentz force action on the hydrostatic balance: the one induced by the magnetic pressure gradient $\nabla P_{\mathrm{mag}} \equiv (1/2\,\mu_0)\,\nabla \boldsymbol{B}^2$ as well as the one induced by the magnetic tension force $\boldsymbol{F}_{\mathcal{L}}^{T} \equiv (1/\mu_0)\,(\boldsymbol{B}\cdot\nabla)\,\boldsymbol{B}$. As shown in Fig. 1 (right), the latter is actually required to counterbalance the effects of the former as the field tends toward a force-free state. It is thus a quantity of importance in the vicinity of the axis of symmetry and at the stellar surface where the field has to be force-free for stability reasons.

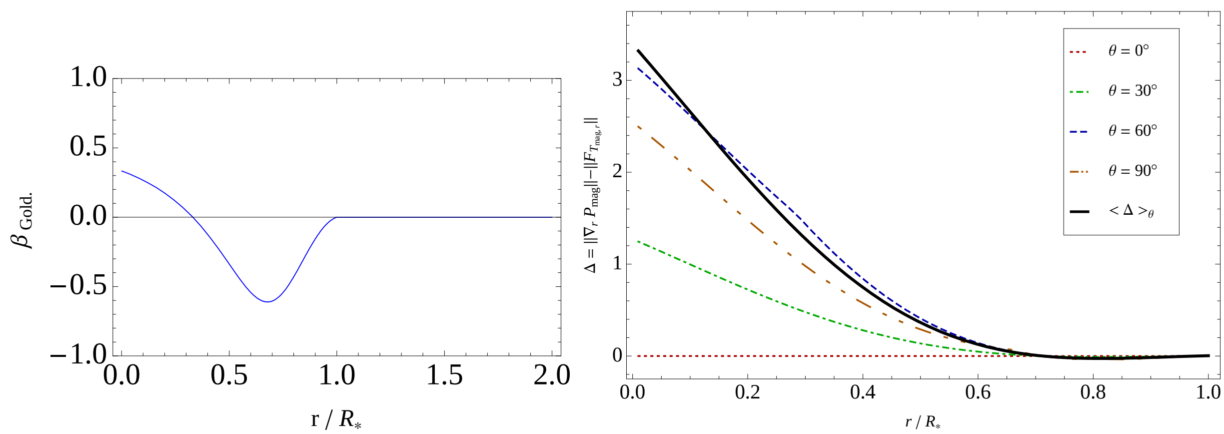

Fig. 1. Left : Amplitude of the parameter $\beta_{\mathrm{Gold.}}$ as a function of the radius. **Right**: Absolute difference Δ between the radial components of the magnetic pressure force and the magnetic tension one (normalized with respect to $B_0^2/\mu_0 R_*$) at several latitudes ($\theta = 0°$, $\theta = 30°$, $\theta = 60°$ and $\theta = 90°$, in dashed lines) and latitudinally-averaged (solid line). Notice that on the axis of symmetry or in the vicinity of the surface, both forces counterbalance each other, leading to a force-free state.

3.2 Perturbations induced on the energetic quantities by a magnetic field on an Ap-star model

The direct contribution of the magnetic field to the change in the energetic balance through ohmic heating or through Poynting's flux is computed by integrating their respective expressions detailed by Duez *et al.* (2008a) over the sphere. It is found that at $r = R_*$ we have $L_* = 3.04 \times 10^{35}\,\mathrm{erg\,s^{-1}}$, whereas the luminosity generated by ohmic heating is $L_\Omega = 5.39\times10^{24}\mathrm{erg\,s^{-1}}$ and the one generated by the Poynting flux is $L_{\mathrm{Poynt}} = 3.15\times10^{25}\mathrm{erg\,s^{-1}}$. The ratio of the classical luminosity over its magnetic contribution is then at the surface $L_*/\left(L_\Omega + L_{\mathrm{Poynt}}\right) = 8.24 \times 10^9$, *i.e.* the energetic perturbations are much weaker than the perturbations generated by the Lorentz force. We can then conjecture that a first approach, consisting in limiting the impact of a large-scale magnetic field only to its impact upon the hydrostatic balance will be justified and that the impact of the magnetic terms on the energetic balance is a higher order perturbation.

3.3 *Perturbations induced on the structural quantities by a magnetic field on an Ap-star model*

A first-order perturbative treatment is performed in the high-β regime ($\beta = \frac{P_{\rm gas}}{P_{\rm mag}}$) to highlight the structural deformations associated with the modification of the hydrostatic equilibrium due to the magnetic field. From our previous remark, the only perturbation taken into account is the one arising from the introduction of the Lorentz force, which is assumed to be weak compared to the gravitational field and to the gaseous pressure gradient. Following Sweet (1950) and Mathis & Zahn (2004, 2005), the equation for the amplitude of the gravitational potential fluctuation $\widehat{\phi}_l$ over the non-magnetic state is derived :

$$\frac{1}{r}\frac{{\rm d}^2}{{\rm d}r^2}\left(r\widehat{\phi}_l\right) - \frac{l(l+1)}{r^2}\widehat{\phi}_l - \frac{4\pi G}{g_0}\frac{{\rm d}\rho_0}{{\rm d}r}\widehat{\phi}_l = \frac{4\pi G}{g_0}\left[\mathcal{X}_{\boldsymbol{F}_{\mathcal{L}};l} + \frac{\rm d}{{\rm d}r}\left(r\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}\right)\right], \tag{3.2}$$

where $\mathcal{X}_{\boldsymbol{F}_{\mathcal{L}};l}$ and $\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}$ are respectively the projections of the radial and the latitudinal $\boldsymbol{F}_{\mathcal{L}}$ components on the Legendre polynomials (cf. these proceedings, "Impact of Large-Scale Magnetic Fields on Solar Structure").
The expressions for the normalized perturbations in gravitational potential $\tilde{\Phi}_l$, density $\tilde{\rho}_l$, pressure $\tilde{P}_l$ ($\tilde{X}_l = \widehat{X}_l/X_0$), and radius c_l have been derived by Mathis, Le Poncin-Lafitte & Duez (2008) and are shown in Fig. 2 (for the modes $l = 0$ and $l = 2$) in the case of an Ap-type star. For kG fields, despite the weakness of the gravitational multipole moment of order 2, it appears that the first-order approach is insufficient to draw any conclusion about the fluctuations in pressure and density at the stellar surface, since these are higher than their unperturbed values owing to the sudden drop of the latter near the surface. However for weaker field, of about 10^2 lower than the value considered here, the treatment is suitable since the fluctuations are proportional to the squared field amplitude. This sudden increase of the perturbation near the surface is in agreement with the value of the plasma parameter β which is lower than unity in this region, indicating that magnetic effects have a crucial impact on the structure of the subsurface layers of such a star. It is then necessary to implement directly the full set of modified equations including the magnetic field in a stellar evolution code.

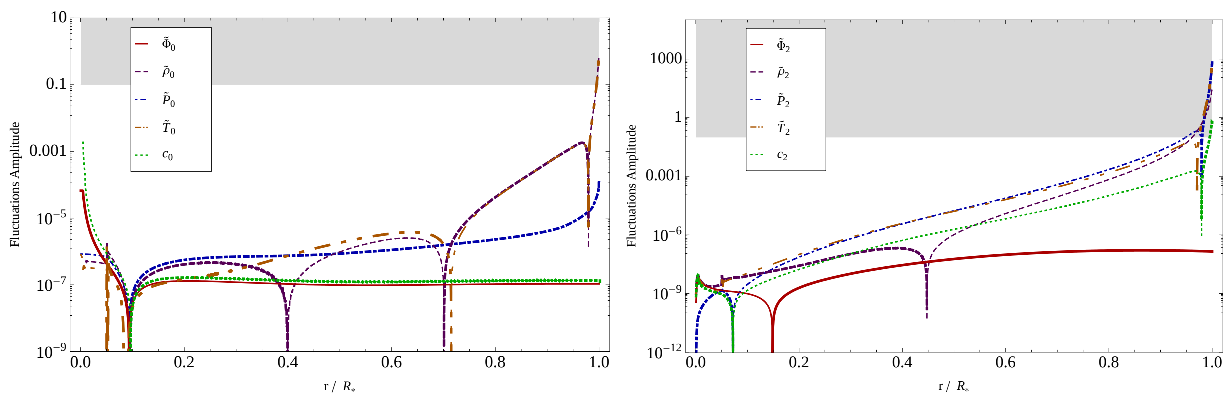

Fig. 2. Normalized modal fluctuations with $l = 0$ (**Left**) and $l = 2$ (**Right**) in gravitational potential, density, pressure, temperature and radius, for a dipolar field in an Ap-type star whose strength at the stellar surface is $B_{\rm surf} = 10$ kG. Bold lines represent negative values. The gray filled area corresponds to the regime where the perturbative approach is invalid an corresponds to the low-β regime.

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