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DISTRIBUTION FUNCTIONS FOR ORBITS TRAPPED AT THE RESONANCES IN THE GALACTIC DISC

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Abstract. The present-day response of a Galactic disc stellar population to a non-axisymmetric perturbation of the potential has previously been computed through perturbation theory within the phase-space coordinates of the unperturbed axisymmetric system. Such an Eulerian linearized treatment however leads to singularities at resonances, which prevent quantitative comparisons with data. Monari et al. manage to capture the behaviour of the distribution function (DF) at a resonance in a Lagrangian approach, by averaging the Hamiltonian over fast angle variables and re-expressing the DF in terms of a new set of canonical actions and angles variables valid in the resonant region. They then follow the prescription of Binney (2016), assigning to the resonant DF the time average along the orbits of the axisymmetric DF expressed in the new set of actions and angles. This boils down to phase-mixing the DF in terms of the new angles, such that the DF for trapped orbits only depends on the new set of actions. This opens the way to quantitatively fitting the effects of the bar and spirals to Gaia data in terms of distribution functions in action space.

Keywords: Galaxy: kinematics and dynamics, Galaxy: disc, Galaxy: structure

1 Introduction

In order to fully exploit the Gaia mission (and spectroscopic follow-ups) data, we need to construct dynamical models of the Milky Way based on distribution functions (DF) – one for each stellar component of the Galaxy and even for the dark matter halo – in self-consistent equilibrium with the gravitational field that they induce. The equation that relates the DFs and the Galactic potential is the collisionless Boltzmann equation (CBE). The Jeans theorem ensures that DFs that depend on the phase-space coordinates only through integrals of motion are solutions of the CBE. Particularly convenient integrals of motion that one can choose are the 'action' variables $\mathbf{J}$ (see Binney & Tremaine 2008), so that the DF is $f_0 = f_0(\mathbf{J})$. The canonical conjugate variables to the actions $\mathbf{J}$ are the angles $\boldsymbol{\theta}$. The equations of motion of stars in the $(\mathbf{J}, \boldsymbol{\theta})$ coordinates are particularly simple, i.e $\mathbf{J} = \text{const}$ and $\boldsymbol{\theta}(t) = \boldsymbol{\Omega} t + \boldsymbol{\theta}_0$, where $\boldsymbol{\Omega}(\mathbf{J}) \equiv \partial H_0/\partial \mathbf{J}$ are the orbital frequencies and H_0 the Hamiltonian function. The actions $\mathbf{J}$ completely characterise a star's orbit, while the the angles $\boldsymbol{\theta}$ the star's phase on the orbit. A DF depending only on the actions represents a phase-mixed system. Using action based DFs, the best axisymmetric models of our Galaxy were constructed (e.g. Cole & Binney 2017).

However, it is nowadays well established that the Milky Way is not axisymmetric, since it contains large non-axisymmetric structures like the bar or the spiral arms. Moreover, the Galactic disc is externally perturbed by its satellites like the Sagittarius dwarf galaxy and the Large Magellanic Cloud. Hence, we require non-axisymmetric DFs to constrain the non-axisymmetric components of the potential, which influence the kinematics of the stars in the solar neighbourhood (see, e.g. Dehnen 1998; Famaey et al. 2005), and act as drivers of the secular evolution of the disc (see, e.g. Fouvry et al. 2015). To take in account non-axisymmetric perturbations in the DF the first step is to linearize the CBE (Monari et al. 2016), discussed here in Sect. 2, and use a special treatment for the orbits trapped at the resonances (Monari et al. 2017), which we discuss in Sect. 3. We conclude in Sect. 4.

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2 Linearisation of the CBE ('Eulerian approach')

The linearisation of the CBE ('Eulerian approach') to the problem posed by non-axisymmetry, developed in Monari et al. (2016), consists in expressing the distribution function of the perturbed system in the action/angle coordinates $(\mathbf{J}, \boldsymbol{\theta})$ of the *unperturbed* axisymmetric system.

Let Φ_1 be the perturbing non-axisymmetric potential, which is always is cyclic in the angle coordinates. We can, therefore, expand Φ_1 in a Fourier series as

$$\Phi_1(\mathbf{J}, \boldsymbol{\theta}, t) = \mathrm{Re}\left\{\mathcal{G}(t)\sum_{\mathbf{n}} c_{\mathbf{n}}(\mathbf{J})\mathrm{e}^{\mathrm{i}\mathbf{n}\cdot\boldsymbol{\theta}}\right\}, \tag{2.1}$$

where $\mathcal{G}(t)$ models the time dependence of the perturbation. In particular, $\mathcal{G}(t) = g(t)h(t)$, where $g(t)$ describe the time dependence of the amplitude of the perturbation, and $h(t)$ sinusoidal function of frequency ω_p, $h(t) = \exp(\mathrm{i}\omega_\mathrm{p} t)$. In particular, if $\omega_\mathrm{p} = -m\Omega_\mathrm{p}$ where m is the multiplicity of the perturber, $h(t)$ describes the rotation of the perturbing potential with a fixed pattern speed Ω_p. The indexes $\mathbf{n}$ run from $-\infty$ to ∞.

Expressing the DF as $f = f_0 + f_1$, where f_0 is the unperturbed axisymmetric DF and f_1 the linear response to the (small) perturbing potential Φ_1, the CBE to the linear order reduces to:

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\partial f_0}{\partial \mathbf{J}} \cdot \frac{\partial \Phi_1}{\partial \boldsymbol{\theta}}. \tag{2.2}$$

Assuming that the amplitude of the perturbation and its time derivatives are null far back in time – i.e., $\forall k, g^{(k)}(-\infty) = 0$ – and that the amplitude of the perturbation is constant at the present time t – i.e. $g^{(0)}(t) = 1$, and $g^{(k)}(t) = 0$, for $k = 1, ..., \infty$ – we can integrate Eq. 2.2 and, as shown in Monari et al. (2016),

$$f_1(\mathbf{J}, \boldsymbol{\theta}, t) = \mathrm{Re}\left\{\frac{\partial f_0}{\partial \mathbf{J}}(\mathbf{J}) \cdot \sum_{\mathbf{n}} \mathbf{n} c_{\mathbf{n}}(\mathbf{J}) \frac{h(t)\mathrm{e}^{\mathrm{i}\mathbf{n}\cdot\boldsymbol{\theta}}}{\mathbf{n}\cdot\boldsymbol{\Omega} - m\Omega_\mathrm{p}}\right\}. \tag{2.3}$$

The linear Eulerian response obtained in this way is valid far away from resonances, but diverges at the resonances, i.e. whenever

$$\mathbf{n}\cdot\boldsymbol{\Omega} - m\Omega_\mathrm{p} = 0. \tag{2.4}$$

Far away from the resonances, we can compute the moments of the perturbed DFs using the linear treatment. For example, Monari et al. (2016), using the epicyclic approximation to express $(\mathbf{J}, \boldsymbol{\theta})$ as a function of the usual positions and velocities $(\mathbf{x}, \mathbf{v})$, and considering 3D spiral arms as the perturber (with corotation in the outer Galaxy) have shown that the spiral arms induce mean radial velocity gradients and vertical motions ('breathing modes') in the Galactic disc in agreement with those found in numerical experiments. Similar gradients and breathing modes have been observed in the extended Solar neighbourhood (Siebert et al. 2011; Williams et al. 2013) (see Fig. 1).

3 Treatment at the resonances ('Lagrangian approach')

The linear Eulerian treatment described in Sect. 2 is valid far from the resonances, where the orbital tori are only distorted by the small perturbing potential Φ_1. But close to resonances, the tori are completely different. For this reason, it is necessary to define an new set of actions and angles to describe the orbits near the resonances (one set for each resonance).

Monari et al. (2017) study this problem in the 2D planar case (but the method can be easily extended to the 3D case). To describe the motion of stars near the resonances it is necessary to pass through two canonical transformations. The first (time-dependent) canonical transformation 'divides' the motion in its fast and slow component. Near a resonance with $\mathbf{n} = (l, m)$,

$$\theta_\mathrm{s} = l\theta_R + m(\theta_\phi - \Omega_\mathrm{p} t), \quad \theta_\mathrm{f} = \theta_R, \quad J_\mathrm{s} = J_\phi/m, \quad J_\mathrm{f} = J_R - (l/m)J_\phi. \tag{3.1}$$

The angle θ_s is slow because, in the unperturbed case, $\Omega_\mathrm{s} \equiv \dot{\theta}_\mathrm{s} \approx 0$. It corresponds physically to the azimuth of the apocentra of the orbit in the reference frame corotating with the perturber. The Hamiltonian of the system can then be averaged over the fast variable, so to reduce the problem to the evolution of the slow angle and action, and making the fast action an approximate integral of motion. Given J_f, $J_\mathrm{s,res}$ is defined as J_s

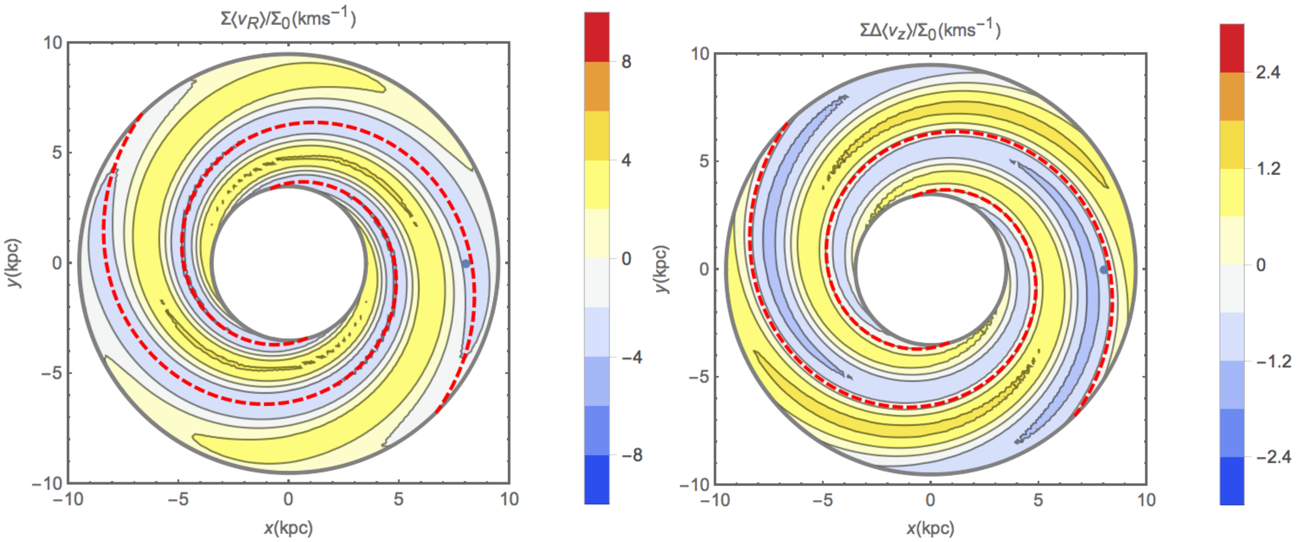

Fig. 1. Mean motions in the Galactic plane caused by 3D spiral arms. Left: mean radial velocity. Right: breathing mode. From Monari et al. (2016).

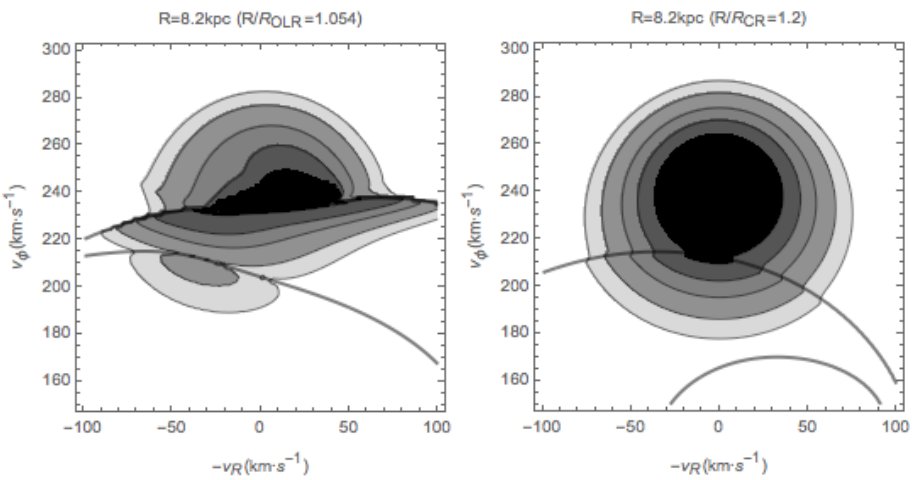

Fig. 2. Velocity distribution functions for stars nearby the Sun for models with fast and slow pattern speed bar and a flat circular velocity curve. The thick lines correspond to zones of trapping to the resonances. Left: $\Omega_\mathrm{b} = 1.8\Omega_0$, nearby outer Lindblad resonance. Right: $\Omega_\mathrm{b} = 1.2\Omega_0$, nearby corotation. From Monari et al. (2017).

where $\Omega_\mathrm{s}(J_\mathrm{s}, J_\mathrm{f}) = 0$. Expanding the averaged Hamiltonian in J_s around $J_\mathrm{s,res}$ near the resonances one obtains a one-dimendional *pendulum* Hamiltonian for the evolution of θ_s. Depending on the energy of the pendulum, this can 'circulate' or 'librate'. In the second case the orbit is trapped to a resonance.

At this point, one needs a second canonical transformation, from the slow angle and action to the actual *pendulum* action and angle $(J_\mathrm{p}, \theta_\mathrm{p})$, to express the DF for trapped orbits as $f_\mathrm{tr}(J_\mathrm{f}, J_\mathrm{p})$. Assuming that the perturbation has been present long enough for phase-mixing the pendulum orbits, a natural choice for f_tr is given byBinney (2016):

$$f_\mathrm{tr}(J_\mathrm{f}, J_\mathrm{p}) = \frac{1}{2\pi}\int_0^{2\pi} f_0(J_\mathrm{f}, J_\mathrm{s}(J_\mathrm{p}, \theta_\mathrm{p}))\mathrm{d}\theta_\mathrm{p} \tag{3.2}$$

where f_0 is the unperturbed DF. In Monari et al. (2017) this method has been applied to study the signature of the bar perturbation on the velocity distribution of stars in the Solar neighbourhood (see Fig. 2).

4 Conclusion

The best way to extract physical information from the Gaia data is to construct action-based dynamical models of the Milky Way. However, it is necessary to take into account the non-axisymmetries in the models and this can be done through perturbation theory. The relevant formalism and methods can be found in Monari et al. (2016) and Monari et al. (2017) and they are also summarised here. In particular, Monari et al. (2016) show how we can linearize the collisionless Boltzman equation away from the resonances and solve it using the using action and angle variables of the unperturbed system. On can in this way evaluate the streaming motions caused by the non-axisymmetries, like the radial velocity gradients and vertical breathing modes caused by spiral arms, similar to those observed in the Solar neighbourhood. Monari et al. (2017) show how a Lagrangian approach allows to describe the DFs for stars trapped at resonances with the perturber, where the Eulerian linear treatment diverges. In this case the motion is described by pendulum action-angle variables and the DF is found averaging the unperturbed distribution function over the pendulum angle. Moreover, the connection with the deformed tori outside of the trapping region is smooth.

Future work on these models will require to move away from the epicyclic approximation to approximate the angle and action variables, and use more general estimates that can take into account eccentric orbits. Other challenges for these models include their extension to the time-dependence of the amplitude of perturbations and to collective effects.

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