SF2A 2018
P. Di Matteo, F. Billebaud, F. Herpin, N. Lagarde, J.-B. Marquette, A. Robin, O. Venot (eds)

STELLAR-MODEL-INDEPENDENT MEASUREMENTS OF γ DORADUS AND SPB INTERNAL ROTATION FROM GRAVITY OSCILLATION MODES.

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Abstract. Owing to the unprecedented quality and long baseline of Kepler photometry, we are finally in a good position to apply asteroseismology to γ Doradus (γ Dor) and Slowly Pulsating B-type (SPB) stars. These intermediate-mass stars pulsate in high radial order gravity modes that probe the deep radiative layers near their convective-core. They are also moderate to fast rotators for which an appropriate treatment of the pulsation-rotation coupling is required to disentangle the oscillation spectrum. On the basis of the traditional approximation of rotation (TAR), we have developed a new stellar-model-independent method to simultaneously estimate the near-core rotation frequency $\nu_{\rm rot}$, the so-called buoyancy radius P_0, and identify the gravity modes. We construct its validity and evaluate its performance on a synthetic spectrum computed from a rotating CESTAM model of a representative γ Dor star. Due to the shortcomings of the asymptotic TAR, we find a slight bias on our estimates of $\nu_{\rm rot}$ and P_0 but we achieve a reasonably good accuracy overall ($\lesssim 6\%$). Finally, we measure the near-core rotation rates in 30 *Kepler* γ Dor stars and compare them with those obtained by another existing method.

Keywords: asteroseismology, stars: oscillations, stars: rotation, methods: data analysis

1 Introduction

Angular momentum (AM) transport processes remain a major uncertainty in the physical description of stars. Two striking examples are the Sun and red giants stars. Indeed, standard rotating 1D stellar models that account for rotationally-induced transport (Zahn 1992; Chaboyer et al. 1995; Maeder & Zahn 1998; Mathis & Zahn 2004) fail to reproduce the interior rotation profile of the Sun, as revealed by helioseismology (e.g. Schou et al. 1998; García et al. 2007; Fossat et al. 2017). In red giants, models predict that the core rotation is a few orders of magnitude faster than what is observed by asteroseismology (Mosser et al. 2012; Gehan et al. 2018). To explain these discrepancies, several additional processes have been suggested involving either internal gravity waves (e.g. Charbonnel & Talon 2005; Fuller et al. 2014) or magnetic fields (e.g. Eggenberger et al. 2005; Rüdiger et al. 2015). However, no clear picture stands out especially about the efficiencies and time-scales of the missing transport mechanisms.

γ Doradus (γ Dor; 1.3-2.0 $M_\odot$) and Slowly Pulsating B-type (SPB; 3-8 $M_\odot$) stars are promising targets to obtain further observational constraints on these processes. These main-sequence stars pulsate in high radial order gravity modes (g modes) that probe the deep radiative layers close to the convective core. Miglio et al. (2008) and Bouabid et al. (2013) demonstrated that the properties of g modes are especially sensitive to the rotation rate and the shape of the chemical gradient at the edge of the convective core boundary.

With pulsation periods of the order of the day, ground-based observations of these pulsators are impractical. In this context, the *Kepler* space mission provided four years of highly-precise and nearly uninterrupted photometry for hundreds of these stars, finally allowing us to undertake their seismic study. However, γ Dor and SPB

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stars are typically in moderate to rapid rotation (Abt et al. 2002; Royer et al. 2007), which significantly affects their oscillation spectrum and has hindered our ability to accurately interpret it. Indeed, in a non-rotating star, g modes are equally spaced in period but this regular structure does not hold in fast rotating stars for which rotation needs to be properly taken into account.

Here, we present a stellar-model-independent method to decipher the oscillation spectrum of rotating γ Dor and SPB stars on the basis of the traditional approximation of rotation (TAR).

2 Method

The traditional approximation of rotation (Eckart 1960; Lee & Saio 1987) treats the rotation-pulsation coupling in a simplified manner to obtain the separability of the pulsation equations while still accounting for the main effects of the Coriolis force. In this approximation, the pulsation equations take a form similar to those of the non-rotating case in the co-rotating frame of reference and the asymptotic analysis of Tassoul (1980) can be applied to obtain a more general expression for the pulsation periods,

$$P^{\mathrm{co}}_{\mathrm{n},\ell,\mathrm{m}} = \frac{P_0\,(n+\epsilon)}{\sqrt{\lambda_{\ell,\mathrm{m}}\,(s)}}, \tag{2.1}$$

where n the radial order, ℓ the angular degree and m the azimuthal order. We choose the convention that $m > 0$ represents prograde modes, and $m < 0$ are retrograde modes. The functions $\lambda_{\ell,\mathrm{m}}\,(s)$ are the eigenvalues of Laplace's tidal eigenvalue problem (see e.g. Unno et al. 1989). They depend on ℓ, m and the spin parameter $s = 2P^{\mathrm{co}}_{\mathrm{n},\ell,\mathrm{m}}\nu_{\mathrm{rot}}$, where ν_{rot} is the star's rotation frequency. The buoyancy radius reads

$$P_0 = 2\pi^2 \left(\int_{\mathcal{R}} \frac{N_{\mathrm{BV}}}{r} \mathrm{d}r \right)^{-1}, \tag{2.2}$$

where N_{BV} is the Brunt-Väisälä frequency and $\mathcal{R}$ is the resonant cavity of the modes. Rotation then lifts the degeneracy in m and the period spacings $\Delta P = P_{n+1,\ell,m} - P_{n,\ell,m}$ are function of the spin parameter. Yet, we highlight the possibility of recovering the equidistance of the spacings by stretching the pulsation periods in the co-rotating frame. Following Eq. 2.1, one may show that, by multiplying the period scale by $\sqrt{\lambda_{\ell,\mathrm{m}}\,(s)}$ where s matches the star's rotation frequency, the associated (ℓ, m) modes are uniformly spaced by P_0.

Taking advantage of this property, we developed a method to seek these new regularities in the oscillation spectrum of γ Dor and SPB stars to infer ν_{rot} and P_0. Details about this stretching method are available in (Christophe et al. 2018). Briefly, we proceed as follows,

1. Extract peak frequencies from the periodogram of the photometric time-series.
2. Pick a guess for the mode identity (ℓ, m) and choose a range of rotation frequencies ν_{rot} to test.
3. For each value of the rotation frequency,
   (a) Switch from the inertial to the co-rotating frame of reference.
   (b) Stretch the pulsation periods.
   (c) Compute the Discrete Fourier Transform (DFT).
4. Stack the DFT spectra obtained on top of another by increasing ν_{rot} to build the DFT map of the parameter space explored (see Fig. 1.b for an illustration).
5. Check if the maximum of Power Spectral Density (PSD) is significant by comparing it to a threshold value of false-alarm probability.
   (a) If significant: identify modes and determine P_0 and ν_{rot} from the maximum of PSD.
   (b) If not: continue the trial and error process by changing the guess for (ℓ, m) or the interval of ν_{rot} tested.

Table 1. Properties of the CESTAM γ Dor model used in Section 3.

$M/M_\odot$	$T_{\rm eff}$ (K)	$\log L/L_\odot$	$\log g$	$R/R_\odot$	Age (Myr)	$X_{\rm C}$	Z	$\langle\nu_{\rm rot}\rangle_{\rm mod}$ (μHz)	$P_{0,\rm mod}$ (s)
1.60	6919	0.7703	4.184	1.69	33.3	0.67	0.0234	35.17	4742

3 Test on a synthetic spectrum

To validate our method, we tested it on a synthetic oscillation spectrum of a representative model of γ Dor star. We used the 1D stellar evolution code CESTAM (Morel & Lebreton 2008; Marques et al. 2013) to calculate a 1.60 $M_\odot$ model on the ZAMS, which main properties are summarised in Table 1. Rotation and standard rotationally-induced transport processes (shear-induced turbulent mixing, meridian circulation) are modelled from the pre-main sequence in a fully consistent manner following the presriptions of Zahn (1992) and Maeder & Zahn (1998). Initial AM content is set assuming a disk-locking model (Bouvier et al. 1997), where, during the Pre-MS, the star is forced to co-rotate with its disk until it dissipates. As shown in Fig. 1.a, the model is in weak differential rotation at the ZAMS, the core rotating slightly faster than the envelope. More details about the stellar model can be found in Ouazzani et al. (2018).

The oscillation modes were computed with the ACOR oscillation code (Ouazzani et al. 2012, 2015), using a non-perturbative method that accounts for both the Coriolis and the centrifugal forces on oscillation modes. Such an approach gives satisfactory results compared to a full 2D treatment as shown by Ballot et al. (2012) and Ouazzani et al. (2017) and has the advantage of requiring less numerical resources. We restrained this test to dipole prograde modes ($\ell = 1, m = 1$) as they are mostly those observed in γ Dor and SPB stars (Van Reeth et al. 2016; Pápics et al. 2017).

In order to compare with the outputs of our method, the buoyancy radius $P_{0,\rm mod}$ and the near-core rotation rate of the model were evaluated from Eq. 2.2 and

$$\langle\nu_{\rm rot}\rangle_{\rm mod} = \frac{\int_{\mathcal{R}} \nu_{\rm rot}(r) N_{\rm BV}(r) \frac{{\rm d}r}{r}}{\int_{\mathcal{R}} N_{\rm BV}(r) \frac{{\rm d}r}{r}}, \tag{3.1}$$

respectively. We applied the stretching method to this synthetic spectrum just as if it was actually observed. Figure 1.b displays the resulting DFT map. Frequencies are correctly identified as ($\ell = 1, m = 1$) modes. The buoyancy radius ($P_0 = 4444$ s) is somewhat underestimated – 6.3% in relative difference – but the rotation frequency ($\nu_{\rm rot} = 37.15$ μHz) remains close to the actual model value – 0.1% in relative difference. Using these estimates, we can build the échelle diagram of the stretched spectrum in periods (see Fig.1.c). If the asymptotic TAR was perfectly suited to describe the mode periods, we would expect a vertical straight line.

This test reveals a slight bias on $\nu_{\rm rot}$ and P_0. For uniformly rotating models, Christophe et al. (2018) showed that a small bias at the level of $\lesssim 5\%$ in relative difference was to be expected for prograde dipole modes due to the shortcomings of both the asymptotic and TAR approximations, independently of the stretching method. In addition, as we assume solid-body rotation, the weak differential rotation of the present model is expected to also impact on our estimates (see also Van Reeth et al. 2018). It almost certainly explains the bending ridge seen in the stretched period échelle diagram.

4 Measurements in *Kepler* γ Dor stars

We determined the near-core rotation rates and buoyancy radii of 30 *Kepler* γ Dors from ($\ell = 1, m = 1$) modes. This sample partially overlaps that of Van Reeth et al. (2016) who used a different method based on fitting model spacings patterns to the observations. Figure 2 compares the results of the two methods. We find a very good agreement on internal rotation rates. Significant disagreement is found under ~ 3700 s for buoyancy radii. Considering the low P_0 of these stars, they are likely evolved on the main sequence so that a chemical composition gradient might exist at the convective core boundary. Such gradient would give rise to a buoyancy glitch (Miglio et al. 2008), making the determinations of P_0 less precise.

Using these measurements, Ouazzani et al. (2018) compared the internal rotation of 36 *Kepler* γ Dor stars to those of models that include rotationally-induced transport. They found that these latter processes cannot explain the measurements, suggesting that an additional AM transport mechanism spins down the core of γ

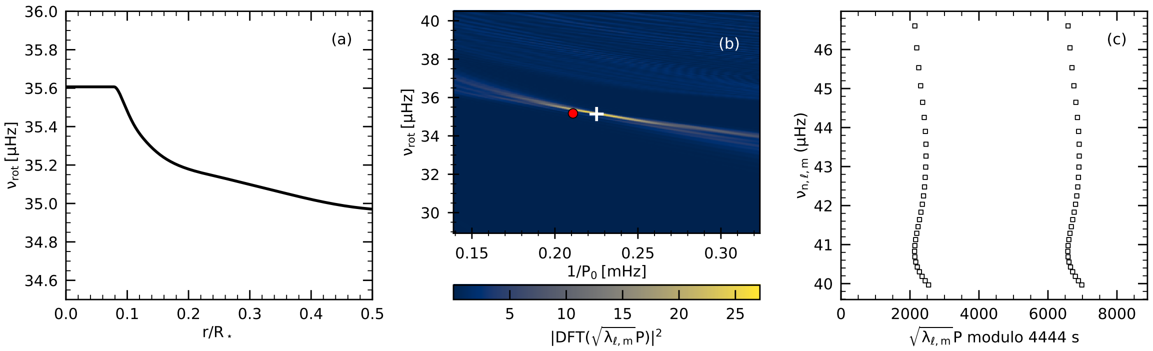

Fig. 1. **(a)**: Rotation profile of the γ Dor model. **(b)**: DFT map obtained by analysing the ACOR oscillation modes computed from this model. Red dot represents the parameter values of the model ($\langle\nu_{\rm rot}\rangle_{\rm mod} = 35.17$ μHz, $P_{0,\rm mod} = 4742$ s). White cross represents the maximum of PSD, i.e. our estimate in a real case scenario ($\nu_{\rm rot} = 35.15$ μHz, $P_0 = 4444$ s). **(c)**: "Stretched" period échelle diagram constructed by using the values of ($\nu_{\rm rot}$, P_0) estimated by the stretching method.

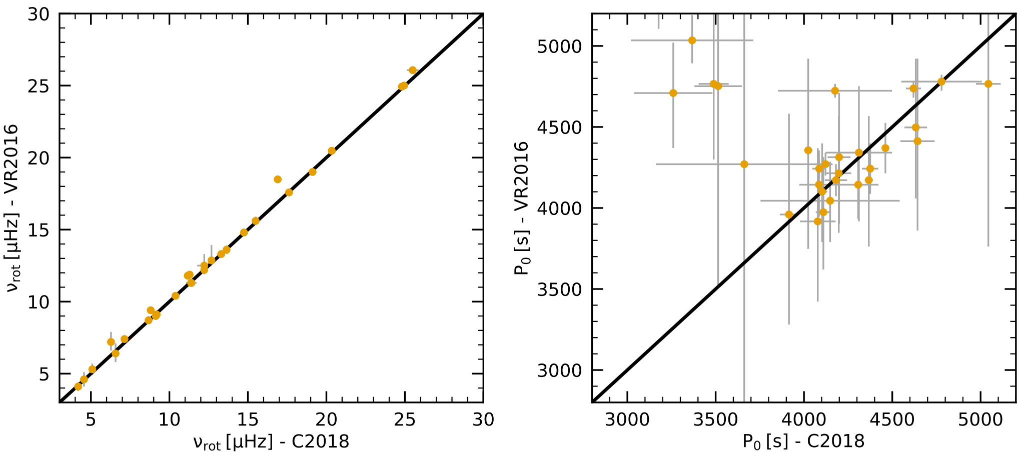

Fig. 2. Comparison between the internal rotation rates (**left**) and the buoyancy radii (**right**) determined by Van Reeth et al. (2016) (VR2016) and those obtained by Christophe et al. (2018) (C2018) using the stretching method.

Dor stars. Whether this mechanism is the same for a red giant star and its progenitor as a γ Dor star remains to be elucidated.

5 Conclusions

We have developed a stellar-model-independent method to interpret the oscillation spectrum of rotating γ Dor and SPB stars. This method allows us to simultaneously obtain the mode identification of g modes and estimate the near-core rotation frequency and stellar buoyancy radius. For a CESTAM γ Dor model in which rotation is treated in a fully consistent manner, we showed that we recover these parameters with a reasonable accuracy (at a few percent level) even in the presence of weak radial differential rotation. We measured the internal rotation rates and buoyancy radius in 30 *Kepler* γ Dor from dipole prograde modes finding good agreement with the model-dependent determinations of Van Reeth et al. (2016) except at low P_0.

Stars of γ Dor type are numerous in the *Kepler* field of view. Including the recent discovery of Rossby modes (Van Reeth et al. 2016; Saio et al. 2018) (that are predicted by the TAR), the study of their oscillation spectra promises to put more stringent constraints on AM transport in stars.

This work was supported by the Programme National de Physique Stellaire (PNPS) of the CNRS/INSU co-funded by CEA and CNES.

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