THE SATURATION OF THE STANDING ACCRETION SHOCK INSTABILITY BY PARASITIC INSTABILITIES

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Abstract.   The Standing Accretion Shock Instability (SASI) is commonly believed to be the cause of large amplitude dipolar shock oscillations observed in numerical simulations of the stalled shock phase of core collapse supernovae. We investigate the role of parasitic instabilities as a possible cause of nonlinear SASI saturation. As the shock oscillations create both vorticity and entropy gradients, we show that both Kelvin-Helmholtz and Rayleigh-Taylor types of instabilities are able to grow on a SASI mode if its amplitude is large enough. We obtain approximate formulae for their growth rates, taking into account the effects of advection and entropy stratification, and use them to estimate the saturation amplitude of SASI. When applied to the set up of Fernández & Thompson (2009), this saturation mechanism is able to explain the dramatic decrease of the SASI power when both the nuclear dissociation energy and the cooling rate are varied. Our results open new perspectives for anticipating the effects, on the SASI amplitude, of the physical ingredients involved in the model of the collapsing star.

1   Introduction

The Standing Accretion Shock Instability (SASI) (Blondin et al. 2003; Scheck et al. 2008) takes place during the first second of the collapse of a massive star, causing the stalled shock to oscillate about the center of the star. 2D simulations suggest that the complex fluid motions triggered by this instability could be a crucial agent of explosion, either by helping the classical neutrino driven mechanism (Marek & Janka 2009; Murphy & Burrows 2008), or by a new mechanism based on the emission of acoustic waves from the proto-neutron star (Burrows et al. (2006), see however Weinberg & Quataert (2008)). The large scale ($l = 1 - 2$) induced asymmetry could also explain the high kick velocities of newly formed neutron stars (Scheck et al. 2006) and may affect their spin (Blondin & Mezzacappa 2007). The degree of asymmetry, however, directly depends on the saturation amplitude of SASI which is not well understood yet. The purpose of this work is to elucidate the saturation mechanism of SASI.

2   Method

We propose that the saturation of SASI takes place when a parasitic instability is able to grow on the SASI mode, and hence feeds upon its energy and destroys its coherence. A parasitic mode can affect the dynamics of SASI if its growth rate $\sigma_{\mathrm{parasite}}$ exceeds the growth rate σ_{SASI} of SASI. Pessah & Goodman (2009) used a similar criterion to estimate the saturation amplitude of the MRI due to parasitic instabilities. We first determine the local growth rate $\sigma_{\mathrm{parasite}}(r, \Delta A)$ of the parasitic instabilities, which is an increasing function of the SASI amplitude ΔA. We then use the criterion $\sigma_{\mathrm{parasite}}(r, \Delta A) = \sigma_{\mathrm{SASI}}$ to define the minimum amplitude $\Delta A_{\mathrm{min}}(r)$ of SASI above which parasites can compete with SASI at a given radius r, despite advection and cooling. The parasitic instabilities can alter the growth of SASI only if their growth takes place in a region which is vital to the mechanism of SASI. For example, if the mechanism of SASI is interpreted as an advective-acoustic cycle (Foglizzo et al. 2007; Foglizzo 2009), this cycle is most sensitive to the region between the shock and the deceleration region where most of the acoustic feedback is produced. Fortunately, as will be shown in Sect. 4,

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the local saturation amplitude displays a broad minimum around the radius $(r_* + r_{\mathrm{sh}})/2$, which defines a global saturation amplitude without much sensitivity on the details of the SASI mechanism.

We also performed non linear simulations measuring the acoustic feedback in the toy model of Sato et al. (2009). These simulations confirmed that the acoustic feedback is decreased when parasitic instabilities are able to grow. The details of the calculations can be found in Guilet et al. (2009).

3 Approximate description of the parasitic instabilities

We estimate the stability of the SASI mode in three steps. First, we study the simplest form of parasitic instability growing on a sinusoidal profile (a velocity profile for the Kelvin-Helmholtz instability, an entropy profile in a gravity field for the Rayleigh-Taylor instability). Second, we assess the stabilizing effect of a positive entropy gradient ∇S_0 in the stationary flow: the restoring force associated to buoyancy can weaken the growth of parasites. In a third step, advection is taken into account: if the fluid is advected fast enough, the instability may be able to grow in a lagrangian way but would actually decay at a fixed radius as the perturbations are advected away. We use numerical simulations to measure the propagation speed of the parasitic instability, by perturbing the SASI mode over a limited region of space (Fig. 1).
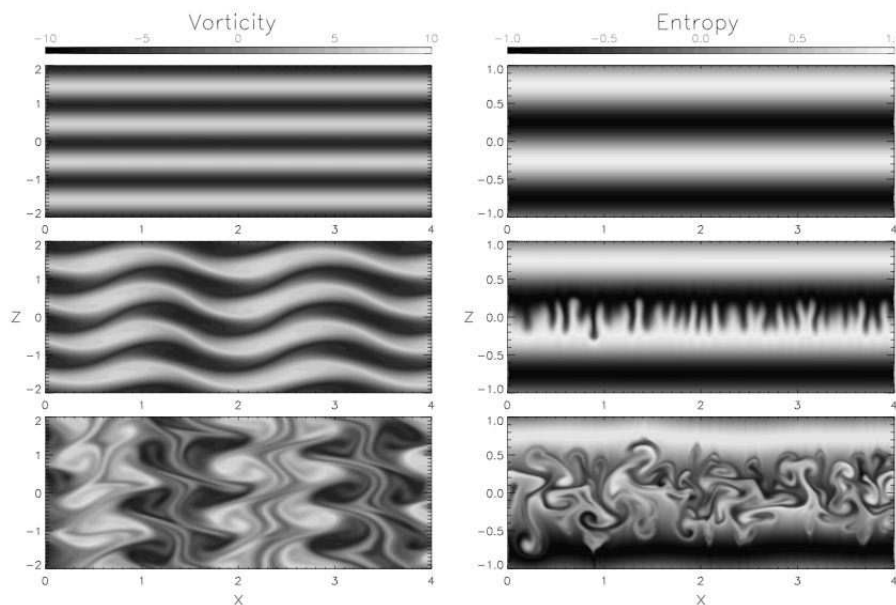
Fig. 1. Different stages in the evolution of the Kelvin-Helmholtz instability (*left*) and the Rayleigh-Taylor instability (*right*). The *upper panel* is the initial condition: a sinusoidal profile of transverse velocity (KH) or entropy (RT) with random perturbations localized around $z = 0$. The *middle panel* shows the time when KH just reached a nonlinear amplitude, and the mode structure can still be recognized. The *bottom panel* shows a more developed nonlinear stage of the instability.

3.1 The Kelvin-Helmholtz Instability

The Kelvin-Helmholtz instability feeds on the kinetic energy available in shear flows. Its growth rate scales like the maximum vorticity Δw, which is here created by SASI. When the stabilizing effects of stratification and advection are taken into account, the maximum KH growth rate can be approximated as :

$$\sigma_{\mathrm{KH}} \simeq \sigma_{\mathrm{0KH}} \left(1 - \frac{\mathrm{Ri}}{\mathrm{Ri}_0} - \frac{v_z}{v_{\mathrm{eff}}} \right), \tag{3.1}$$

where $\sigma_{\mathrm{0KH}} \sim 0.25 \Delta w$. The Richardson number Ri is the squared ratio of vorticity to the Brunt-Väisälä frequency and $\mathrm{Ri}_0 \sim 0.24$ is the critical value for the stabilization by stratification. Finally $v_{\mathrm{eff}} \sim 1.5 \Delta w/K$ is an effective propagation velocity of KH along the vertical direction.

3.2   The Rayleigh-Taylor Instability

The Rayleigh-Taylor instability feeds on the potential energy available when a low entropy fluid is sitting on top of a higher entropy one, in a gravitational acceleration g. We estimate the RT growth rate in the presence of advection and stratification as:

$$\sigma_{\mathrm{RT}} = 0.75 \sqrt{\frac{\gamma - 1}{\gamma} g \nabla \left(\Delta S + S_0 \right)} - 0.6 K v_z, \tag{3.2}$$

where ΔS is the entropy amplitude of the SASI mode, S_0 is the dimensionless entropy of the stationary flow, K is the SASI wave number and v_z is the advection speed.

4   Comparison with the simulations of Fernandez & Thompson (2009)

In order to test this scenario, we have applied the above estimates to the setup of Fernández & Thompson (2009). Their simulations studied the effect of iron dissociation by removing a fixed energy per nucleon at the shock. This dissociation energy ϵ is varied in a parametric manner from zero to $\epsilon = 0.25 v_{\mathrm{ff}}^2$ (v_{ff} is the free fall velocity at the shock), while the cooling function is adjusted in order to keep constant the stationary shock radius. These simulations are a very good test for any saturation mechanism because the saturation amplitude of SASI was found to be sensitive to the parameter ϵ.

The local saturation amplitude is minimum at an intermediate radius between the proto-neutron star and the shock (roughly at $r_{\mathrm{mini}} \sim (r_{\mathrm{sh}} + r_*)/2$). The reason is that just below the shock parasites are efficiently stabilized by advection, while close to the proto-neutron star they are strongly stabilized by the entropy stratification (Fig. 2, left). The most efficient growth of the parasites therefore takes place where neither advection nor stratification is strong. If the amplitude is larger than this quite flat minimum, the parasite can grow in a large region of the flow around the radius r_{mini}. This minimum is thus a meaningful estimate of the amplitude at which the saturation by the parasites occurs (whether it be RT or KH).
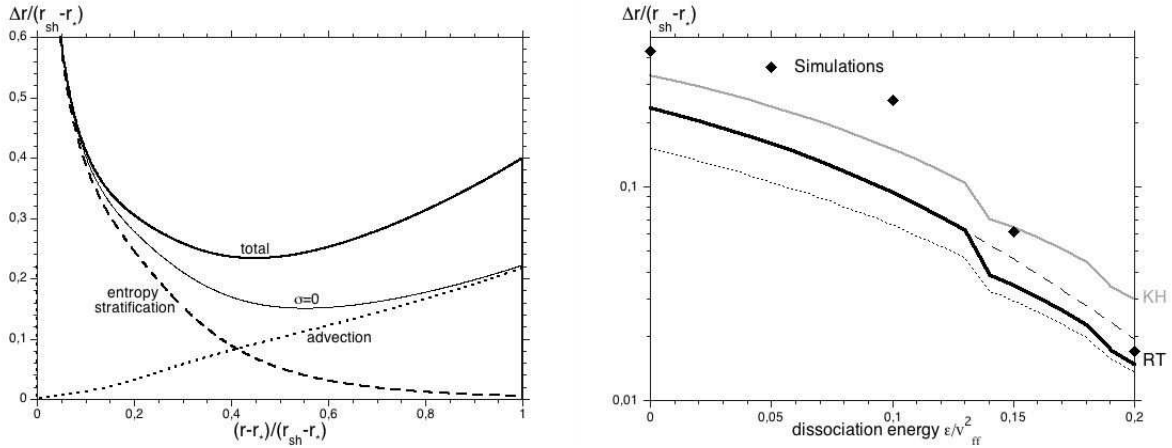
Fig. 2. *Left:* Effect of advection, entropy stratification, and SASI growth rate, on the "local saturation amplitude" of the shock displacement $\Delta r(r)$. This plot describes the saturation of the fundamental mode of SASI by the growth of the RT instability, when $\epsilon = 0$. The "local saturation amplitude" (thick line) is also shown when the growth rate of SASI is neglected (thin line). The dotted and the dashed lines show the contributions of advection and entropy stratification. The results are qualitatively the same if one considers the KH instability, or higher harmonics of SASI, or $\epsilon > 0$. *Right:* As a function of the dissociation energy ϵ, comparison of the saturation amplitude measured in the simulations by Fernández & Thompson (2009) (diamonds) with those deduced from RT (black thick line) and KH (gray thick line) instabilities growing on the most unstable SASI mode. The effect of neglecting the growth rate of SASI (dotted line), or considering only the fundamental mode of SASI (dashed line), is also shown.

The saturation amplitude predicted by our analysis of KH and RT instabilities is compared with the results of the simulations by Fernández & Thompson (2009) in the right panel of Fig. 2. The RT instability appears to

grow at smaller SASI amplitudes than KH, and is thus expected to be the dominant parasitic mode. We note that the movies published online by Fernández & Thompson (2009) show mushroom-like structures, consistent with our conclusion that RT is the dominant secondary instability.

The trend of a strong amplitude decrease with increasing dissociation is reproduced by the parasitic instability saturation mechanism. Furthermore our estimate of the saturation amplitude is consistently $15 - 50\%$ below the simulated value at all ϵ. Given the uncertainties due to our approximate description of the parasites and the many other non linear effects we neglected, and given that we expected to underestimate the saturation amplitude, the agreement is very encouraging.

We can take advantage of the analytic nature of our description to understand the causes of this decrease. The dissociation energy loss at the shock leads to more compression, hence a slower advection speed and a faster propagation of parasitic instabilities against the flow. The saturation amplitude is decreased by a factor 4.5 between $\epsilon = 0$ and $\epsilon = 0.2$, due to the faster growth of RT. A change in the flow profile due in particular to the decrease of the entropy stratification is also responsible for a factor 2 decrease. Two other effects are responsible for a moderate 25% decrease : the decrease of the SASI growth rate, and the fact that the dominant SASI mode becomes dominated by higher harmonics (which are more sensitive to parasites).

5   Conclusion

We have developed the first predictive mechanism for the saturation of SASI, in which a parasitic instability growing on the SASI mode causes the saturation. The two potentially important instabilities, Rayleigh-Taylor and Kelvin-Hemhotz, have been characterized taking into account the effect of entropy stratification and advection. The saturation amplitude of a SASI mode has been estimated by comparing its growth rate with that of the parasitic modes. This estimate reproduces the decrease of the SASI power with dissociation energy observed in the simulations of Fernández & Thompson (2009).

If confirmed, our results would open new perspectives for anticipating the effect on the SASI amplitude of other physical ingredients ignored here such as a realistic equation of state, the neutrino-heating rate, and the rotation and magnetic field of the progenitor star. They could also be useful as an input for analytical models studying the possible consequences of SASI, such as the model for gravitational wave emission proposed by Murphy et al. (2009).

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