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FROM SOLAR TO STELLAR OBLATENESS

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Abstract. Rotation, and more precisely differential rotation, have a major impact on the internal dynamics of stars (and the Sun) and induce many instabilities driving the transport of angular momentum. In the present paper we shall consider the effects on the shape of shellular layers, and to first order, these concerning the apparent oblateness. The first case concerns the Sun, for which accurate limb fluctuations permit to ascertain not only the oblateness, but also its shape, that it to say the determination of the radius with latitude. Thanks to the advent of interferometry techniques, stellar shapes can be now measured with a great accuracy. We will review here the main results obtained so far on different stars and we will give their physical parameters derived from a model fitting procedure. It will be shown how the core density can be reached.

1 Introduction

The non homogeneous mass and velocity rate distributions of stars, including the Sun, modifies their outer shape. Up to a recent date, this departure to sphericity has been considered only as a second order effect on theories of stellar structure. However, discrepancies between models and observations have been noticed, so that the study of stellar shapes cannot be bypassed anymore (Meynet, 2009).

With the advent of more and more precise observations through dedicated techniques such as interferometry, the signature of stellar oblateness, that is to say the difference between the equatorial and polar radius Δr, is now acknowledged. The question is to incorporate this parameter in the equations presently used in stellar interior models to compute the effects on central density.

We will first recalled the solar case, in order to see to what extent the physical basis reached for the internal solar structure can be applied to stars.

2 What can be learned from solar oblateness?

One of the puzzling features of the solar fundamental parameters is its oblateness, as the way to estimate it strongly depends on the variation of both the velocity rate of the rotation, non uniform in latitude and depth, and the distribution of mass inside the Sun. If we look at the Sun as a succession of shellular layers, they all have a different density $\rho(r)$ and they all rotate with a different angular velocity. It is known today that the solar core ($0 < r < 0.2 \ \ R_\odot$) is rigidly rotating faster than the surface, maybe nearly twice (Sturrock, 2009). The tachocline $0.713 < r < 0.718 \ \ R_\odot$) plays a key role in differentiating the rotation up to the near surface. The thin width of this layer is not supposed to be independent of latitude. In the case of a slowly rotating star (like the Sun), it has been shown that hydrostatic background acquires a small ellipticity (as the angular velocity profile is viscously dominated). Charbonneau et al. (1999) found that the mean position of the tachocline moves upwards with latitude. They found a shift of $(0.024 \pm 0.004)R_\odot$ in tachocline position between the latitudes of 0° and 60°, a result found again by Basu & Antia (2006). At the top of the convective zone, the leptocline ($0.975 < r < 0.995 \ \ R_\odot$) will change the shape of the outer surface, mainly due to a reversal

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	Spectral Type	Mass ($M_\odot$)	b ($R_\odot$)	a ($R_\odot$)	T_{eff} (K)	References
Alderamin	A7 IV-V	2.0	2.823	2.175	4750	van Belle et al., 2006
Achernar	B3 Vpe	6.07	12.0	8.3	15000	de Souza et al., 2008
Altair	A7 IV-V	1.8	1.915	1.681	7680	van Belle et al., 2001
Vega	A0 V	2.303	2.873	2.306	9306	Peterson et al., 2006
Regulus	B7 V	3.4	4.16	3.14	12901	McAlister et al., 2005
υ **Cygni**	B2 Ve	6.81	4.62	3.93	19600	Neiner et al., 2005
Rasalhague	A5 III	3.0	2.871	2.390	8250	Zhao et al., 2007

Table 1. Fundamental parameters as deduced from literature, for the seven stars for which an oblateness was observed (up to 2008).

	Alderamin HD 203280	**Achernar** HD 10144	**Altair** HD 187642	**Vega** HD 172167	**Regulus** HD 87901	υ**Cygni** HD 202904	**Rasalhague** HD 159561
$\omega(\mu rad/s)\sin i$	157	36	196	153	118	112	161)
$T_c(K)$	8.2310^{6}	2.6010^{7}	1.3210^{7}	1.6310^{7}	2.2610^{7}	3.4410^{7}	1.4510^{7}
$P_c(Pa)$	2.0110^{15}	6.5510^{13}	6.4710^{15}	2.3510^{15}	1.2610^{15}	2.8610^{15}	3.8110^{15}
$\rho_c(g/cm^3)$	11.87	0.52	30.04	12.45	6.44	8.35	15.67
J_2	-8.1310^{-3}	-0.1010^{-2}	-4.5910^{-2}	-7.1110^{-2}	-8.6110^{-2}	-5.5210^{-2}	-6.1410^{-2}
Oblateness	0.229	0.308	0.122	0.197	0.245	0.149	0.167
A(gr/cm^2)	4.8810^{55}	2.4810^{57}	2.2410^{55}	6.0110^{55}	1.7710^{56}	4.8210^{56}	8.0510^{55}
C(gr/cm^2)	6.1310^{55}	3.3610^{57}	2.5410^{55}	7.3110^{55}	2.2510^{56}	5.5910^{56}	9.5110^{53}

Table 2. Derived Stellar Parameters from our study.

of the gradient of the radial velocity rate : $\partial\Omega/\partial r$ being < 0 from the equator to around 60°, then canceling and becoming > 0 beyond. The whole shape remains oblate, varying in phase with solar activity, albeit the different layers radius just beneath the surface shows a non monotonic expansion with time (in antiphase with solar activity, the strongest variations of the stratification located at around 0.995 $R_\odot$) (Lefebvre and Kosovichev, 2005). Lefebvre et al. (2009) have studied the behavior of key physical parameters in this layer, for instance opacities changes, super-adiabatic stratification, hydrogen and helium ionisation processes. Probably this layer is the cradle of in-situ changes of magnetic fields, which will need further studies.

From such physical grounds, it can be easily understood that the limb fluctuations reflects the properties of all the physical mechanisms from the core to the surface. If we are able to quantify the chain, it would be possible, from accurate measurements of the outer shape, to go down to the core. For the Sun, we are progressively paving the way, and we are just touching the point to understand how the different mechanisms are articulated all together (Turck-Chièze and Mathis, 2009). Obviously dedicated astrometric space missions (i.e. experiments working at the mas level of accuracy) already scheduled to be launched in a near future (such as SDO –Solar Dynamics Observatory–), will help in a need future to solve such question.

3 Modeling star oblateness

The first approach was made by Chandrasekhar (1933) who was able to compute oblateness of stars under the assumption of a non differential rotation and a power density law with the radius r of the Star. The problem was taken again, mainly by Tassoul (2000). It is shown that rotation flattens the star and produce non uniform temperature and density distributions (i.e. gravitational darkening, De Souza et al, 2002). While the flattening mostly increases the absolute flux level of the energy distribution, gravitational darkening makes an equatorial viewed star apparently cooler than a star seen through the pole.

We will assume here a Clayton's model (Clayton, 1986) for which developments are given elsewhere (Damiani et al., 2010). The main point is the introduction of a scale-length parameter a in the behavior a the pressure gradient with the radius variation r from the core to the surface. a vanishes at the center to increases rapidly in absolute magnitude, until reaching the radius where the density $\rho(r)$ begins to decline where it flattens out, to latter on asymptotically decline toward 0. Thus a has a physical meaning and becomes a key parameter to determine the pressure variation with r, and then the central density. This model must be used cautiously but it can be refined a little bit more by taking into account the oblateness. If we look at the inverse problem, the observed oblateness will permit to derive central density and also the moment of inertia C and A on the z and x axis respectively, and from them, the gravitational moment J_2 which is directly linked.

4 Results

An observed oblateness of seven stars have been reported in the literature. Table 1 gives their spectral type, mass M, effective temperature T_{eff}, as well as their polar b and equatorial a measured radius. From these data, calculated parameters are listed in Table 2: central temperature T_c, central pressure P_c and central density ρ_c, and certainly for the first time the A and C moments of inertia and the gravitational second order moment J_2. This last one seems to be faint, in absolute value, four to five orders of magnitude less than the solar estimate (which is $\approx -2 \times 10^{-7}$).

5 Conclusion

Crucial new insight into the stellar properties is gained from the observations of oblate stars indicating a clear need for more sophisticated stellar structure models than current "standard" models. We encourage observations of oblate stars in order to determine their equatorial and polar radius, through existing facilities such as the CHARA array, the Keck interferometer, the Navy prototype Interferometer (NPOI) or the Palomar Testbed Interferometer PTI). A catalogue of 67 prospective rotationally distorted stars has been given by Van Belle et al (2004) who gave a rough estimate of the ratio R_b/R_a based upon a simplification of an expression describing self-gravitating rotationally distorted gaseous masses: $vsini \approx (2GM/R_b \times (1 - R_b/R_a))^{0.5}$. In the case of Altair, the approximation gives 1.14 instead of 1.16 observed.

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