*SF2A 2009*
*M. Heydari-Malayeri, C. Reylé and R. Samadi (eds)*

# SIMILARITY CONCEPTS AND SCALING LAWS OF THE ACCRETED COLUMN IN MAGNETIC CATACLYSMIC VARIABLES: THE POLAR PROJECT

Falize, E.[1,3], Loupias, B.[1], Dizière, A.[2], Ravasio, A.[2], Gregory, C.D.[2], Cavet, C.[3], Michaut, C.[3], Koenig, M.[2], Leidinger, J.P.[2], Ribeyre, X.[4], Nazarov, W.[5], Barroso, P.[3], Millerioux, M.[1], Chevrot, M.[2] and Leconte, L.[2]

**Abstract.** In this paper we present the similarity properties and scaling laws of the accreted column in magnetic cataclysmic variables. To model this astrophysical object, two radiating regimes have been explored and different classes of scaling laws have been obtained.

## 1 Introduction

Magnetic cataclysmic variables (mCVs) are close binary systems containing an accreting magnetic white dwarf which accretes matter from a late type Roche-lobe filling secondary star (Warner 1995). The presence of intense magnetic field, radiation and hydrodynamics imply a rich range of behavior at different spatial and time scales. Moreover it leads to a variation of observable quantities and a variety of observed optical, X-ray and radio phenomena. These astrophysical objets class provides an excellent probe of the magnetic accretion processes under extreme conditions. More generally, the cataclysmic variables constitute an important class of low-luminosity, compact galactic X-ray sources. This radiation mainly comes from an area near the white dwarf surface: the accreted column (Kylafis & Lamb 1982; Chevalier & Imamura 1982; Cropper 1990; Wu 2000) which has a thousand kilometers spatial extension (Wu et al. 1994; Falize et al. 2009b). It is currently admitted that mCVs is divided in two sub-classes according to the intensity of magnetic field: the intermediate polar or DQ Her star which has $B < 10$ MG, and the polar ($B > 10$ MG) or Am Her star. In the last sub-class the Alfven radius is greater than the L1 Lagrange point; consequently the magnetic field is enough strong to prevent the formation of an accretion disc which is present in non magnetic or weakly magnetic cataclysmic variables. It imposes the white dwarf to rotate synchronously, to guide the flows and it could also dictate the radiative loss processes (Warner 1995). Having a correct model of the radiating region allows to determine fundamental properties of the mCVs (Suleimanov et al. 2005). The possibility to study the dynamics of the accreted column by a new way is very important in order to test the theoretical model. The intense development of laser facilities, which allows to bring up matter to extreme states of density and temperature in laboratory, is a very promising way for astrophysicists to explore the coupling between radiation and matter (Remington et al. 2006). In this paper we present recent results which show that we can reproduce *exactly* the accreted column in laboratory (Falize et al. 2008; Falize et al. 2009b). In the first part, we present an extended classification of similarity concepts that we use in laboratory astrophysics. In the second part we remind the accreted column standard model, we analyze the similarity properties and establish the scaling laws of the accreted column in different radiating regimes. Finally we conclude on the results.

[1] CEA-DAM-DIF, F-91297 Arpajon, France
[2] LULI Ecole Polytechnique, CNRS, CEA, UPMC, Route de Saclay, 91128 Palaiseau, France
[3] LUTH, Observatoire de Paris, CNRS, Universit Paris-Diderot, 92190 Meudon, France
[4] Centre Lasers Intenses et Applications (CELIA), UMR 5107 Université Bordeaux 1, 351, cours de la Liberation, 33405 Talence Cedex FRANCE
[5] School of Chemistry, University St Andrews, Purdie Blg, St Andrews KY16 9ST, United Kingdom

## 2 Symmetry and Invariance concepts

Symmetry provides tools to analyze the solutions of a problem without needing to solve the problem explicitly (Olver 1995) or it can also suggest methods to simplify it. In order to assure the link between the experimental and astrophysical systems we must elaborate scaling laws which will allow us to characterize the astrophysical character of experimental plasma. Although the time and spatial scales are very different between the astrophysical phenomena and the laboratory plasma, we can exhibit a theoretical connexion (scaling laws) which assures equivalent physics. Consequently scaling laws play a major role in laboratory astrophysics in order to connect the astrophysical phenomena or object to laboratory plasmas but also in order to connect the laboratory plasmas created with different powerful facilities. Laboratory astrophysics experiments can be divided into several fundamental classes which are connected to the type of similarity used as can be seen on the following diagram 1b.

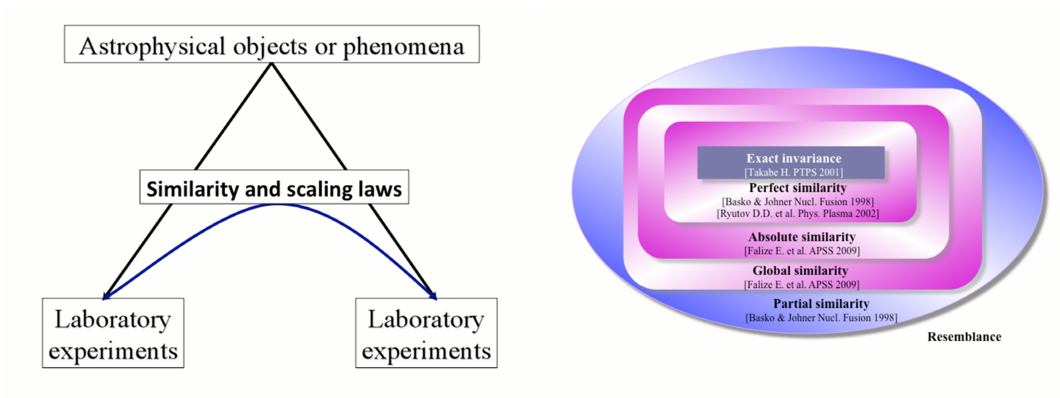


**Fig. 1.** a. Not only are the scaling laws fundamental in order to connect the astrophysical phenomena to laboratory plasmas but also in order to connect the laboratory plasmas themselves. b. The diagram of experiments classification.

These classes are:

1. **Exact invariance**: also called sameness (Takabe 2001), includes the experiments which consist in reproducing thermodynamical conditions identical to those of the astrophysical objects. More generally these experiments do not introduce the notions of time and space and allow to determine the input data of hydrodynamic models such as the opacity or equations of state of matter in extreme conditions.

2. **Similarity**: Although the spatial and temporal scales are very different, the astrophysical relevance of these experiments can be checked thanks to scaling laws to assure that the physical system under study satisfies similarity properties. This type of experiments gathers, for instance, high Mach numbers plasmas (Loupias et al. 2007; Gregory et al. 2008). This specific class can be decomposed in four sub-classes:

   (a) *Perfect similarity*: It is a concept introduced by Ryutov & Remington (2003) and this type of scaling laws are firstly identified in hydrodynamics by Basko & Jonher (1998). This type of scaling laws consists in rescaling the time and spatial variables only.

   (b) *Absolute similarity*: This scaling law class is very interesting in order to adapt a target design to powerful facilities or to rescale a system with the same composition (Falize et al. 2009a).

   (c) *Global similarity*: Based on the Lie symmetry invariance, the global scaled invariance only needs the invariance of the equation form. It is less constraining that the absolute similarity and introduces additional free parameters in order to rescale astrophysical experiments (Falize et al. 2009a) as we will see in the next section.

   (d) *Partial similarity*: The principle of this last scaling law category is to conserve only a part of equation and examine the effect of other physical phenomena on different quantity evolutions. This principle has been used by Basko & Jonher (1998) in the context of inertial fusion.

3. **Resemblance**: This last category gathers all laboratory astrophysics experiments which allow to verify a physical model or astrophysical numerical simulations. In other words, this category joins the experiments which depend greatly on microscopic physics.

## 3 Similarity properties and scaling laws of the accreted column

### 3.1 Accreted column: the standard model

When the supersonic in-falling matter, which is canalized by the magnetic field, impacts the white dwarf photosphere, a shock front is created. The kinetic energy of the material in the accretion stream, acquired from the Roche lobe, is converted into radiation. The equilibrium height $x_s$ of the shock is determined by the requirement that the post-shock flow must have enough time to cool in order to match the conditions in the stellar photosphere. Consequently $x_s$ is determined by the efficiency of cooling processes (bremsstrahlung, cyclotron and compton cooling) which act in the accreted column. The relative importance of these cooling processes depends mainly of the magnetic field B and accreted the mass rate $\dot{m}_a$. In order to model this specific zone of in-falling matter, we consider a plane-parallel, collisional shock with a post-shock medium where the local gravitational field does not modify its structure and which can be described successively by a one (Kylafis & Lamb 1982; Falize et al. 2009b) and two temperature regimes (Saxton & Wu 1999).

### 3.2 One-temperature radiating plasmas scaling laws

In the one-temperature approximation, the dynamics of the accreted column is given by:

$$\frac{d\rho}{dt} + \rho\frac{\partial v}{\partial x} = 0, \quad \frac{dv}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial x} = 0, \quad \frac{dP}{dt} - \gamma\frac{P}{\rho}\frac{d\rho}{dt} = -(\gamma-1)\mathcal{L}(\rho, P) \tag{3.1}$$

where we have respectively the mass conservation, the impulsion conservation and the equation of energy. The quantities $x$, $t$, $\rho$, $v$, $P$, $T$, $\gamma$, $\mathcal{L}$ are respectively the spatial and temporal coordinate, the density, the velocity, the pressure, the temperature, the adiabatic index and the cooling function. In this model we suppose a cooling function in the form[1] $\mathcal{L}(\rho, P) = \mathcal{L}_0\rho^\epsilon P^\zeta$, and we consider the plasma as a perfect gas, *i.e.* $P = \varepsilon_0\rho T$. Using the homothetic group we build the scaling laws which are given by:

$$\rho = a\tilde{\rho}, \quad x = b\tilde{x}, \quad P = c\tilde{P}, \quad \varepsilon_0 = d\tilde{\varepsilon}_0 \tag{3.2}$$

$$t = b\sqrt{\frac{a}{c}}\tilde{t}, \quad v = \sqrt{\frac{c}{a}}\tilde{v}, \quad T = a^{-1}cd^{-1}\tilde{T}, \quad \mathcal{L}_0 = a^{-(\epsilon+1/2)}b^{-1}c^{3/2-\zeta}\tilde{\mathcal{L}}_0 \tag{3.3}$$

where the quantities with the tilde correspond to the laboratory quantities. We see that we have four free parameters in the global similarity case (a, b, c, d) and only two in the absolute similarity case (a,b) with $\mathcal{L}_0 = \tilde{\mathcal{L}}_0$ and $\varepsilon_0 = \tilde{\varepsilon}_0$ (Falize et al. 2009a). To both similarity results, we add the analytical solution which describes the structure of the cooling layer in a stationary regime (Falize et al. 2009b). It allows to determine the spatial extension of the cooling layer corresponding to 100 $\mu$m in the laboratory field if the cooling time is 1 ns and the shock velocity is 100 km/s. Consequently the structure of the cooling layer and its dynamics can be diagnosed allowing potentially to analyze the quasi-periodic oscillation phenomena in laboratory. This result has given birth to the POLAR experiment which has been recently realized with LULI2000 facility. The goal of this first experiment was to determine the similarity properties of laboratory plasmas and the measurements are still being analyzed. To go deeper in the similarity analysis we have been examining the two-temperature regime presented below. In order to complete the scaled analysis we examined the two temperature regime that we present now.

[1] Let's note that in the bremsstrahlung case $\epsilon = 3/2$ and $\zeta = 1/2$.

### 3.3 Two-temperature radiating plasmas scaling laws

When $t_{ei} > t_{cool}$ where $t_{ei}$ and $t_{cool}$ are respectively the characteristic time of energy exchanged by Coulombian collision and the cooling time, the temperature of the electron decreases faster more rapidly than the collisions heat the electron (Kylafis & Lamb 1982; Wu 2000). Thus it is necessary to include the two-temperature effects and in order to describe its evolution we use the standard model described by Saxton & Wu (1999) and we add the energy equation:

$$\frac{dP_e}{dt} - \gamma \frac{P_e}{\rho}\frac{d\rho}{dt} = (\gamma - 1)[\Gamma(\rho_i, \rho_e, T_e, T_i) - \Lambda(\rho_e, T_e)] \tag{3.4}$$

where $P_e$, $T_e$, $T_i$ and $\rho_e$ are respectively the electron pressure, the electronic and ionic temperature and the electronic density. The heating due to the difference of temperature is given by:

$$\Gamma(\rho_i, \rho_e, T_e, T_i) = \Gamma_0 \rho^2 \left[\frac{T_i - T_e}{(T_e + m_e T_i/m_i)^{3/2}}\right] \tag{3.5}$$

where $m_e$ and $m_i$ are respectively the electron mass and the ion mass. We suppose a cooling function in the form $\Lambda(\rho_e, P_e) = \Lambda_0 \rho_e^{\alpha} P_e^{\beta}$. By using the same theoretical formalism as in the one-temperature case, we construct the scaling laws of two-temperature plasmas:

$$\rho = a\tilde{\rho}, \quad \Lambda_0 = b\tilde{\Lambda}_0, \quad \Gamma_0 = c\tilde{\Gamma}_0, \tag{3.6}$$

$$x = a^{(7-6\beta-4\alpha)/(2\beta+1)} b^{-4/(2\beta+1)} c^{(3-2\beta)/(2\beta+1)} \tilde{x}, \quad t = a^{(5[1-\beta]-3\alpha)/(2\beta+1)} b^{-3/(2\beta+1)} c^{2(1-\beta)/(2\beta+1)} \tilde{t},$$

$$v = a^{(2-[\alpha+\beta])/(1+2\beta)} b^{-1/(1+2\beta)} c^{1/(1+2\beta)} \tilde{v}, \quad P = a^{(5-2\alpha)/(1+2\beta)} b^{-2/(1+2\beta)} c^{2/(1+2\beta)} \tilde{P},$$

The important result is the existence of one free parameter in absolute similarity (b=c=1) which allows to scale such plasmas in laboratory. Although we consider here the regime in the accreted column context, the applicability of these results is fundamental to several astrophysical environments.

## 4 Conclusion

In this paper we presented different results on the similarity properties of the accreted column in magnetic cataclysmic variables. We have seen that reproducing exactly the astrophysical phenomena and examining its behaviors and its dynamics in details is theoretically possible. It is the aim of the POLAR experiment project. The possibility to reproduce these phenomena is a real opportunity to increase our understanding of the physics of accreted processes.

## References

Basko, M. M. & Johner, J. 1998, Nucl. Fusion, 38, 1779

Chevalier, R. A., & Imamura, J. N. 1982, ApJ, 261, 543

Cropper, M. 1990, Space Sci. Rev., 93, 611

Falize, E., et al. 2009a, ApSS, 322, 107

Falize, E., et al. 2009b, ApSS, 322, 71

Falize, E., et al. 2008, J. Conf. Phys. Ser., 112, 042016

Gregory, C. D., et al. 2008, ApJ, 676, 420

Kylafis, N. D., & Lamb, D. Q. 1982, ApJS, 48, 239

Loupias, B., et al. 2007, Phys. Rev. Lett., 99, 265001

Olver, P. J. 1995, Equivalence, Invariants, and Symmetry, Cambridge

Remington, B. A., Drake, R. P. & Ryutov, D. 2006, Rev. Modern Phys. 78, 755

Ryutov, D. D., & Remington, B. A. 2003, Phys. Plasma, 10, 2629

Saxton, C. J., & Wu, K. 1999, MNRAS, 310, 677

Suleimanov, V., Revnivtsev, M., & Ritter, H. 2005, A&A, 435, 191

Takabe, H. 2001, Progress of Theoretical Phys. Suppl., 143, 202

Warner, B. 1995, Cataclysmic variable stars, Cambridge Univ. Press, Cambridge

Wu, K. 2000, Space Sci. Rev., 93, 611

Wu, K., Chanmugam, G., & Shaviv G. 1994, ApJ, 426, 664