# SOME RESULTS ON DISTURBANCE REJECTION CONTROL FOR AN ADAPTIVE OPTICS SYSTEM

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**Abstract.** Using linear quadratic gaussian (LQG) control theory, we propose a disturbance rejection control for an adaptive optics (AO) system. An a posteriori frequency analysis of the AO multivariable feedback system is carried out to check stability and robustness properties. We present numerical simulations to demonstrate the effectiveness of the proposed approach.

Keywords: adaptive optics, linear quadratic gaussian control, MIMO feedback analysis

# 1 LQG disturbance rejection control for an AO system

# 1.1 LQG state-space system

Deformable mirror (DM) and wavefront sensor (WFS) dynamics are assumed linear and determined by their influence matrices and pure delays (command input, measured output) as in Kulcsár et al. (2000). An autoregressive (AR) system describes the time evolution of the atmospheric wavefront. The obtained LQG discrete-time state-space system is diagonal and separates the plant dynamics (DM & WFS) and the disturbance dynamics (AR model). Thus, the AO control problem can be formulated as a LQG disturbance rejection control problem, see Bitmead et al. (1990); Folcher et al. (2010).

# 1.2 Control objectives & LQG design

Good adaptive optics performance (resulting in high Strehl ratios in the data) is obtained when the residual wavefront variance is weak. Keeping DM command input in an admissible range is also an important control objective. These two specifications can be translated in terms of the LQG cost criterion (to be minimized) for the given state-space system. The solution of the LQG problem, called the LQG controller, is simply the combination of a linear quadratic regulator (LQR) and a linear quadratic estimator (Kalman filter). The separation principle (see Kwakernaak and Sivan (1972); Anderson and Moore (1990)) guarantees that optimal state feedback gain and optimal observer gain can be computed independently. Moreover, for this specific disturbance rejection problem, the gains computation involve the resolution of reduced order Algebraic Riccati equations. This makes this control problem amenable to an efficient numerical solution.

# 2 Numerical simulations

# 2.1 Main parameters

For numerical modeling purposes, the Software Package CAOS Carbillet et al. (2005) is used to generate  $1000 \times 1 \text{ ms}$  wavefronts propagated through an evolving 3-layers turbulent atmosphere ( $r_0=10 \text{ cm}$  at  $\lambda=500 \text{ nm}$ ,  $\mathcal{L}_0=25 \text{ m}$ , wind velocities=8–16 m/s). We consider an 8-m telescope, with 0.1 obstruction ratio. Wavefronts are projected over a Zernike polynomials base of size 15. DM controls perfectly low spatial frequencies: the influence matrix is  $M_m = I_{n_b}$ . An 8×8 ( $\Rightarrow$ 52) subaperture Shack-Hartmann WFS (8×8 0.2" px/subap.,  $\lambda_0=700 \text{ nm}$ ) is choosen. The WFS influence matrix  $M_w$  was previously determined numerically (interaction matrix computed through simulation of the AO system calibration).

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#### 2.2 LQG controller design

The sampling period is T=1 ms, and command input and measured ouptut delays are considered unitary. The weighting matrice R, which defines the minimized LQG quadratic cost is fixed to  $R=10^{-2}$ . Two WFS noise levels are considered:  $V=10^{-2}I$  (design 1) and  $V=10^{-4}I$  (design 2), for which we obtain two candidate controllers. We invite the interested reader to consult the companion paper Folcher et al. (2010) for more details.

#### 2.3 Frequency analysis of the AO multivariable feedback system

The residual wavefront variance control objective can be written in the frequency domain for given atmospheric wavefront's power spectral density, and WFS noise's power spectral density. This relation (see Kulcsár et al. (2000)) gives constraints on the frequency response of the residual wavefront rejection transfer function and the measurement noise rejection transfert function. The singular value plot of these highly multivariable transfer functions are given in the companion paper Folcher et al. (2010) for the two candidate controllers. Design 2 rejects better the residual wavefront, but is more sensitive to measurement noise. Singular values of residual wavefront rejection transfer function also exhibits a higher resonant factor which indicates a weak input stability margin. The frequency analysis selects the controller of design 1 as the final controller.

#### 2.4 Time responses

Three components of the signal  $w_a$  (Zernike coefficients from the simulated atmospheric wavefronts) and residual wavefront coefficients  $w_r$  are plotted in Fig. 1. LQG controller rejects the atmospheric perturbation: residual wavefront coefficients  $w_r$  are reduced by a factor of 100.



Fig. 1. Time evolution of  $w_a$  (on the left) and  $w_r$  (on the right) for the 5th mode (plain line), for the 10th mode (dashed line), and for the 15th mode (dashed-dot line).

As a result of this test simulation: on the 15 Zernike modes controlled, the total standard deviation drops down from  $\sim 1030 \text{ nm}$  to  $\sim 30 \text{ nm}$  (all modes standard deviation:  $\sim 1150 \text{ nm}$ , uncorrected modes:  $\sim 500 \text{ nm}$ ).

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