A NEW MODEL OF COMETARY NON-GRAVITATIONAL FORCES

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Abstract. The gravitational orbit of a comet is affected by the sublimation of water molecules by the nucleus when the comet approaches perihelion. This outgassing triggers a non-gravitational force (NGF) which significantly modifies the orbit of the comet. The amplitude of the perturbation depends on several parameters which can be constrained by visible, infrared and radio observations of the coma and nucleus of the comet. It depends also on the nucleus density, which can in turn be determined by modeling the effect of the NGF on the orbit of a comet. This method is the only method available so far to estimate the density of cometary nuclei. Up to now, the modeling of this effect is mostly based on an empirical model defined in the early 70's which uses a simplified isotropic outgassing model. Attempts have been made to use advanced anisotropic thermal models to calculate the NGF taking into account several observational constraint and to retrieve the nucleus density, but: (i) this approach is restricted to a handful of cometary nuclei which are sufficiently well-known to allow this type of modeling, and (ii) the authors usually don't fit directly the astrometric measurements but rather non-gravitational parameters calculated with the above-mentioned empirical model. We present a new model for non-gravitational forces with the aim of revisiting the problem of NGF calculation and nucleus density determination. The method is based on the separation of the surface of the nucleus in several surface elements located at different latitudes. The contribution of each surface element to the overall NGF is fitted from the astrometric measurements together with the density of the nucleus. This new method will be used to interpret future astrometric measurements of these pristine objects with GAIA.

Keywords: non-gravitational forces, comets, dynamics

1 Introduction

The last empirical model of non-graviational forces mostly used is Marsdens'one (Marsden et al. 1973) which was developped about 37 years ago. This symetrical model considers a global activiy of an half pure ice nucleus.

In this present work, we are revisiting the problem of NGF calculation and nucleus density determination. In the first section, we present the hypothesis of the model and a geometrical aproach. Then we focus our study on the calculation and the fit of the model. Finally, the last section is dedicated to the preliminary results.

2 Hypothesis of the model

The nucleus is modelized as a pseudo-sphere cut into latitudinal bands (Fig. 1). The thermal inertia of the nucleus is neglected and the gas velocity is considered proportional to the thermal gas velocity. These hypotheses are less restrictive than the Marsden's one (Marsden et al. 1973) and allow the seasonal effects.

3 Calculation and fit of the model

After simplification of the calculation, we see that the acceleration due to the outgassing of the water depends on the product of density by the radius of the nucleus ρR and on the coefficients C_i describing the activity of each latitudinal bands :

$$\overrightarrow{A_{NG}}(t) \propto \frac{1}{\rho R} \sum_{i=1}^{k} C_i \left(\frac{d\overrightarrow{F}(t)}{dS} \right)_i \tag{3.1}$$

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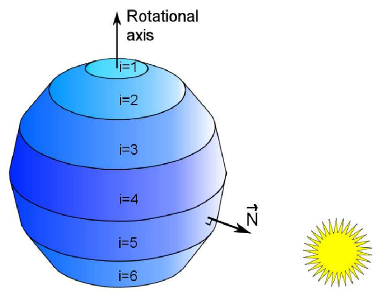

Fig. 1. Geometrical view of a modelized nuclei with seven latitudinal strips.

The surfacic force (Fig. 2) depends on the sublimation rate Z from the incidence of the sun rays and the calculation of the gas velocity V_g from the temperature (Crifo 1987) :

$$\left(\frac{d\overrightarrow{F(t)}}{dS}\right)_i = Z_i(t).V_{g_i}(t).M_{H_2O}.\overrightarrow{N_i} \tag{3.2}$$

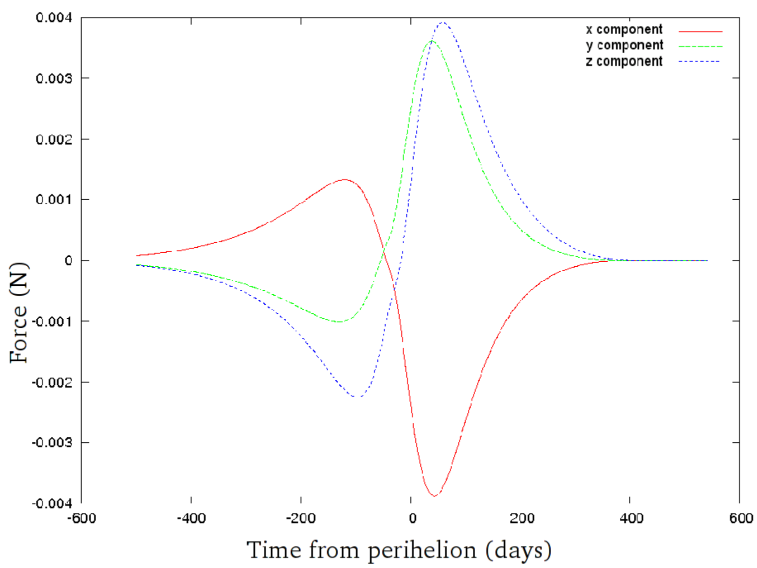

Fig. 2. Example of force components of one of the bands.

The fitting of the model was made by least square method and now the L-BFGS method (a quasi-Newton method for constrained optimisation) is being tested. The fitted parameters are :

- the initial conditions of position and velocity of the comet
- ρR, the density-radius
- coefficients C_i (between 0 and 1)

The fitted data are :

- currently : astrometric positions from the MPC
- In a near future : water production rate (HST, radio measurements in Nancay and also visible observations), gas velocity (radio in Nancay)

4 Preliminary results

4.1 Verification of the calculation of the subsolar point

The latitude of the subsolar point is the starting point for calculating the elevation of the sun, the gas production rate and thus non-gravitational forces. Its position in the cometocentric frame varies over time depending on the rotational axis inclination, and on the nucleus rotation around its axis and around the sun. Figure 3 shows that the calculation of the latitude of the subsolar point gives similar results than Davidsson and Gutiérrez (2004).

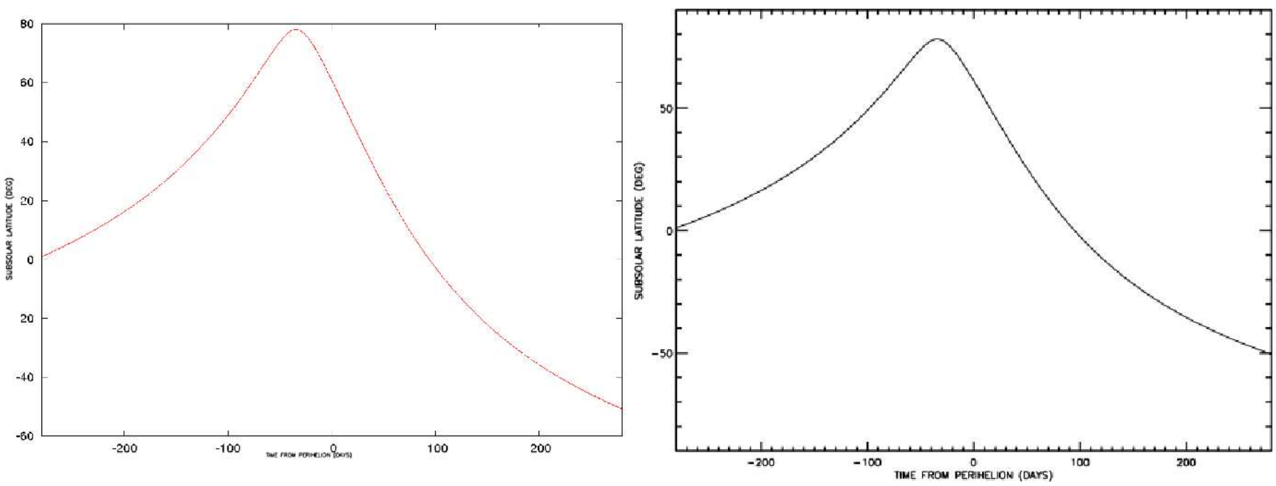

Fig. 3. Latitude of the subsolar point with axis orientation $(\alpha, \delta) = (220^\circ, -10^\circ)$ compared to the calculation of Davidsson and Gutiérrez (2004) on the right for the comet 19P/Borelly

4.2 Calculation of the non-gravitational acceleration

Figure 4 shows the gas production curve. Which is obtained from the following energy balance :

$$(1 - A)\,\frac{F\odot}{R_h^2}\cos z = \epsilon\sigma T^4 + H(T)Z(T)(1-\alpha) \tag{4.1}$$

where A is the Bond's albedo, $F\odot$ the solar constant, R_h the heliocentric distance of the comet, z the elevation of the sun, ϵ the infrared emissivity of the nucleus, σ the Stefan-Boltzmann constant, T the surfacic temperature of the zone, α a water recondensation parameter (Crifo 1987), $H(T)$ the sublimation latent heat and $Z(T)$ the sublimation rate. The displacement of the peak from perihelion is due to the spin axis orientation (,)=(220,-10).

Figure 5 shows the maximal contributions of each latitudinal band to the non-gravitational acceleration for a seven strips model. We can see that all the contributions are relatively distinct hence allowing an adjustement of the coefficients C_i. The total non-gravitational acceleration consists in a linear combination of these contributions.

5 Conclusions

In a near future, the model will be fitted by the Broyden-Fletcher-Goldfarb-Shanno method to constrain the C_i coefficients. We will apply this model to known comets like, 19P/Borelly, 81P/Wild2, 103P/Hartley2 and 1P/Halley. One of the main goal is to determine the density-radius. GAIA astrometric measurements will be very useful as they will allow to measure comet nucleus position.

The next evolutions of the model will be to use an ellipsoidal shape for the nucleus as well as to add the fitting of the production rate using $Af\rho$ measurements and of the gas velocity using radio measurements from Nancay observatory.

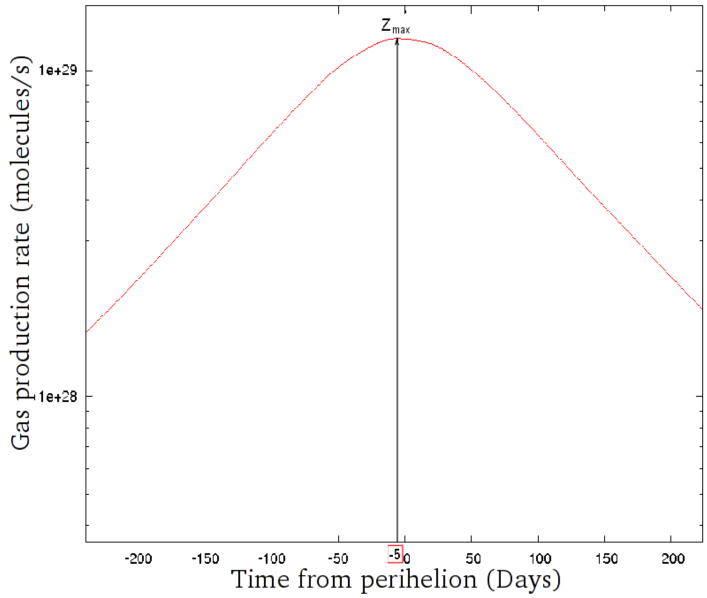

Fig. 4. Gas production rate of a fully active nucleus with spin axis orientation $(\alpha, \delta) = (220^\circ, -10^\circ)$ (Davidsson and Gutiérrez 2004) as function of time for the comet 19P/Borelly

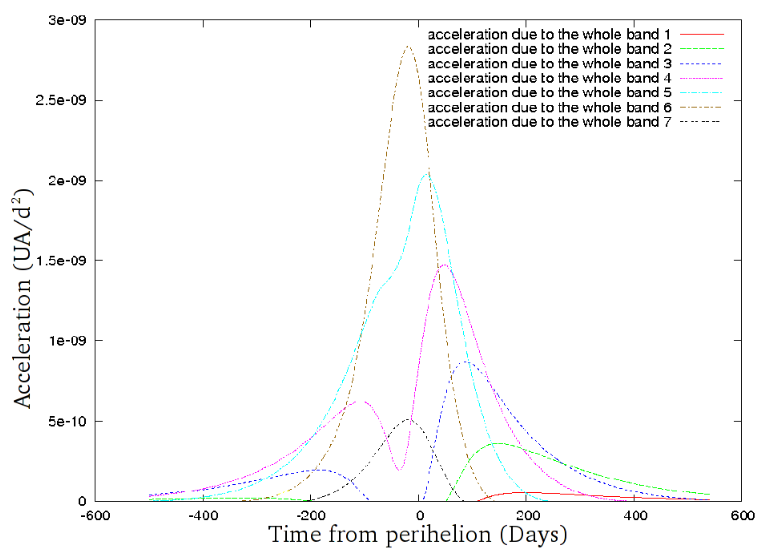

Fig. 5. Non-gravitational accelerations of the whole bands for a seven strips model of the comet 19P/Borelly

References

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