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COMPRESSIBLE TURBULENCE IN ASTROPHYSICS

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Abstract. It is well known that compressible turbulence is a dominant feature of interstellar clouds; it also plays a non negligible role in the evaluation of the local heating in the solar wind. Nevertheless due to the intrinsic difficulty to include compressible effects in theoretical models almost no serious result exists in compressible turbulence. In this note, we report a recent progress made on compressible isothermal turbulence in the asymptotic limit of a high Reynolds number. Our investigation concludes on the existence of a double inertial range where respectively super- and sub-sonic turbulences happen.

Keywords: Turbulence, solar physics, interstellar medium

1 Introduction

Almost all important results in turbulence concern incompressible fluids (see *e.g.* Frisch 1995). In his third 1941 turbulence paper Kolmogorov derived an exact relation for incompressible isotropic hydrodynamics in terms of third-order longitudinal structure function in the asymptotic limit of a high Reynolds number (Kolmogorov 1941). Because of the rarity of such results the Kolmogorov's universal (4/5s) law has a cornerstone role in the analysis of turbulence: for example, it is the starting point for the prediction of the well known $E(k) \sim k^{-5/3}$ energy spectrum. Extensions have been made of such a result to astrophysical magnetized fluids described in the framework of MHD (Politano & Pouquet 1998) as well as Hall MHD (Galtier 2008).

The previous results are found for incompressible fluids and no universal law of that type has been derived for compressible turbulence which is far more difficult to analyze. The lack of knowledge is such that even basic statements about turbulence like the presence of a cascade, an inertial range and constant flux energy spectra are not well documented. This fact hinders the domain of application of compressible turbulence which ranges from aeronautical engineering to astrophysics. In the latter case, it is believed that highly compressible turbulence controls star formation in interstellar clouds (Elmegreen & Scalo 2004).

In the general case, our knowledge of compressible hydrodynamic turbulence is mainly limited to direct numerical simulations (see *e.g.* Passot et al. 1988; Federrath et al. 2010). The most recent results for supersonic isothermal turbulence with a grid resolution up to 2048^3 (Kritsuk et al. 2007) reveal that the inertial range velocity scaling deviates substantially from the incompressible Kolmogorov spectrum with a slope of the velocity power spectrum close to -2 and an exponent of the third-order velocity structure function of about 1.3. Surprisingly, the incompressible predictions are shown to be restored if the density-weighted fluid velocity, $\rho^{1/3}\mathbf{u}$, is used instead of simply the velocity $\mathbf{u}$. Although a -2 spectrum may be associated with shocks – like in one dimension – it seems that their contribution in three dimensions is more subtle. Generally speaking it is fundamental to establish the equivalent of the 4/5s law for compressible turbulence before going to the more difficult problem of intermittency.

In this note we discuss some recent results on compressible isothermal turbulence obtained by Galtier & Banerjee (2011).

2 Compressible isothermal turbulence

The starting point of our analysis is the three-dimensional compressible hydrodynamics equations which are often used in direct numerical simulations for investigating interstellar turbulence (Kritsuk et al. 2007; Federrath

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et al. 2010). In a recent paper (Galtier & Banerjee 2011), we have derived the equivalent of the Kolmogorov law for compressible structure functions. The main difference with the incompressible case resides in the presence of a new type of term which acts on the inertial range similarly as a source or a sink for the mean energy transfer rate. When isotropy is assumed, compressible turbulence may be described by the relation

$$-\frac{2}{3}\varepsilon_{\rm eff}\, r = \mathcal{F}_r(r)\,, \tag{2.1}$$

where $\mathcal{F}_r$ is the radial component of the two-point correlation functions and $\varepsilon_{\rm eff}$ is an effective mean total energy injection rate. By dimensional arguments we predict that a spectrum in $k^{-5/3}$ may still be preserved at scales smaller than the sonic scale $1/k_S$ if the density-weighted fluid velocity, $\mathbf{v} \equiv \rho^{1/3}\mathbf{u}$, is used for the spectrum instead of simply the velocity $\mathbf{u}$. However, at super-sonic scales $\varepsilon_{\rm eff}$ becomes space dependent and a $k^{-19/9}$ spectrum may be predicted dimensionally. This result is summarized in Fig. 1: it is a schematic view of the density-weighted energy spectrum. Two regimes are expected according to the scale with a super-sonic turbulence regime at the largest scales and a sub-sonic one at scales smaller than the sonic scale $1/k_S$. This result illuminates some recent high-resolution direct numerical simulations where this behavior was observed (Kritsuk et al. 2007; Federrath et al. 2010).

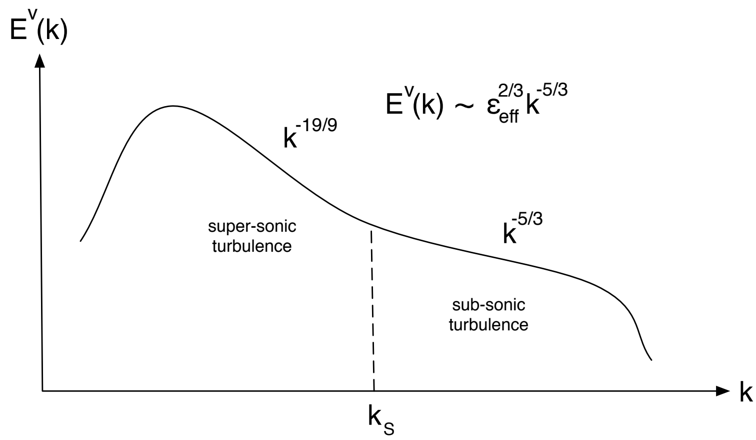

Fig. 1. Schematic view of the spectrum for compressible isothermal turbulence. k_S is the sonic scale.

3 Perspective

We believe that our results on compressible isothermal turbulence are also important in the context of recent observations made with the Herschel telescope (Arzoumanian et al. 2011) which reveal some universality in molecular clouds. Indeed, the interstellar filaments seem to exhibit a universal width of the order of the sonic scale whose origin remains unknown. As proposed by the authors, the answer is certainly hidden in compressible turbulence.

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