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# THE ROSSBY WAVE INSTABILITY IN 2D VISCOUS PROTOPLANETARY DISCS

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**Abstract.** To form meter-sized pre-planetesimals in protoplanetary discs, dust aggregates have to collide at velocities lower than a threshold so that they are not destroyed during the collision. Dust grains are affected by the gas through aerodynamic drag, therefore its study is relevant in determining the grain growth evolution. The Rossby Wave Instability (RWI, non-axisymetric instability, Lovelace et al. 1999; Lovelace & Hohlfeld 1978), may solve this problem by the formation of vortices where dust particles can accumulate, collide at low velocity and stick together (Barge & Sommeria 1995). We study the development of the RWI in a 2D viscous disc using the Smoothed Particle Hydrodynamics (SPH) method. We show that Rossby Vortices can form and survive in the disc for a few thousand years. The instability is triggered by a "*jump*" in the radial surface density profile caused by the presence of a dead zone in the protoplanetary disc.

## 1 Introduction

The formation of preplanetesimals is the first step of planetary formation. Current models and experiments of grain growth do not explain how grains can reach sizes larger than $\approx 1$ m. The existence of the "radial-drift barrier" introduced by Weidenschilling (1977), the "fragmentation-barrier" and the "boucing-barrier", presented by Güttler et al. (2010) and Zsom et al. (2010), are the current limits of the first steps of planetary formation. Barge & Sommeria (1995) suggested the existence of vortices in protoplanetary discs that could develop and exist for hundreds of years. In their center, grains could accumulate and stop their inner migration, leading to easier growth thanks to smaller relative velocities. Indeed, it is well known that dust grains tend to accumulate in pressure maxima. This condition can be satisfied if the gas density is locally higher as in anticyclonic vortices. The Rossby Wave Instability may solve this problem as it generates Rossby Vortices where dust particles can accumulate. Lyra et al. (2008, 2009) underline the fact that vortices trap the dust in their center, creating embryos of various sizes in approximately a hundred years.

We will first briefly describe the Rossby Wave Instability (RWI) and its trigger mechanism. Then we present the Smoothed Particle Hydrodynamics (SPH) approach, the $\alpha$ artificial viscosity and numerical setup. Section 4 details the results obtained from the 2D simulation of gas only, and conclusions of this work are presented in Section 5.

## 2 The Rossby Wave Instability

The Rossby Wave Instability in protoplanetary discs was first introduced by Lovelace et al. (1999), after the previous work of Lovelace & Hohlfeld (1978). They presented the necessary criterion to develop the instability (see Section 2.1). Tagger (2001) presented an analytical study of this instability reminding the fact the Vortices should be destroyed by the differential rotation of the nebula. He showed that the RWI can be maintained thanks to the coupling between density waves and Rossby waves at the corotation radius. Numerical simulations performed by Li et al. (2000) showed that a "*bump*" or a "*jump*" in the radial density profile can trigger the instability. Varnière & Tagger (2006) used this method showing that an overdensity can naturally result from the presence of a dead zone (DZ) in the disc. The accretion is slower in the DZ than in the inner or outer part of the disc, therefore (1) at the DZ's outer edge, gas coming from the outer disc accumulates and (2) at the DZ's inner edge, the gas in the DZ is accreted more slowly than matter present in the inner disc. Finally both configurations produce an overdensity such that the RWI can develop.

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### 2.1 Instability criterion

A necessary criterion to develop the RWI in the disc is to have an extremum of the quantity $\mathcal{L}$ (Lovelace et al. 1999; Lovelace & Hohlfeld 1978):

$$\mathcal{L} = \frac{\Sigma\Omega}{\kappa^2}\frac{P}{\Sigma^\gamma} = \left(\frac{(\nabla \times \mathbf{v})_z}{\Sigma}\right)^{-1} \quad \text{for an isothermal disc,} \tag{2.1}$$

where $\Omega$ is the orbital time, $\kappa$ the epicyclic frequency, $\gamma$ the adiabatic index, $P$ the pressure, $\Sigma$ the surface density and $\mathbf{v}$ is the gas velocity.

## 3 SPH simulations

We use a 2D SPH code (Barrière-Fouchet et al. 2005) that models an unmagnetized protoplanetary disc containing only gas around a one-solar mass star. We implement a dead zone (DZ) to examine how SPH simulations can develop the RWI. The DZ is created by introducing a radial dependance in the $\alpha$ artificial viscosity profile. We study the resulting effects on the surface density profile $\Sigma(r)$.

### 3.1 $\alpha$ artificial viscosity

To run the simulations, we use the standard artificial viscosity of Gingold & Monaghan (1983) introduced to avoid interpenetration of SPH particles during shocks and parametrized by two parameters: $\alpha$ and $\beta$. The $\alpha$ parameter is linked to the Shakura & Sunyaev (1973) $\alpha_{\rm ss}$ parameter (Meglicki et al. 1993), which represents the turbulent viscosity of the gas and is responsible for the mass accretion rate onto the central star.

### 3.2 Disc model

The disc extends from 0.1 AU to 15 AU and the dead zone from 1 AU to 5 AU. The temperature and surface density profiles follow $\Sigma \propto r^{-p}$ and $T \propto r^{-q}$ with $p$=3/2, $q$=3/4. The initial state is made of 500,000 gas particles in a 2D near-equilibrium disc that relaxes to a stationary disc in a few years. We let the disc evolve over 1600 years. The standard artificial viscosity is parametrized by $\alpha = \alpha(r)$ and $\beta = 0$ (Section 3.1). The code units are: 1 $M_\odot$, 7.5 AU, $(7.5)^{3/2}$ yr, $G$=1.

### 3.3 Dead Zone profile

To create an artificial dead zone in the disc, we introduce a radial variation in the $\alpha$ artificial viscosity (Section 3.1). We use the profile of Varnière & Tagger (2006) so that the viscosity varies like:

$$\begin{cases} \alpha = a_0\left(\epsilon + \mathrm{atan}(\delta_r(-r + r_{\rm in}^{\rm DZ}))\right) & \text{if } \ r \le r_{\rm in}^{\rm DZ}, \\ \alpha = a_0\epsilon & \text{if } \ r_{\rm in}^{\rm DZ} < r < r_{\rm out}^{\rm DZ}, \\ \alpha = a_0\left(\epsilon + \mathrm{atan}(\delta_r(r - r_{\rm out}^{\rm DZ}))\right) & \text{if } \ r \ge r_{\rm out}^{\rm DZ}, \end{cases} \tag{3.1}$$

where $r_{\rm in}^{\rm DZ} = 1$ AU is the inner edge of the dead zone, $r_{\rm out}^{\rm DZ} = 5$ AU is the outer edge of DZ, $\delta_r = 50$ is the slope of the viscosity profile at the edges of the DZ, $\epsilon = 10^{-5}$ is the residual viscosity in the DZ and $a_0 = 10^4$. Figure 1 gives the representation of the artificial viscosity profile in the disc.

## 4 Results from gas simulations - Evidences of the RWI

### 4.1 Density fluctuations

Figure 2 shows the density fluctuations $(\rho - \langle\rho\rangle)$ at four different times during the disc evolution. At early times, the viscosity profile does not permit to have a strong enough maximum in the density profile. The accumulation of gas at the edges of the dead zone produces an overdensity satisfying the $\mathcal{L}$-criterion. At early times, the simulation shows the formation of many vortices. After about 100 years, they merge in five dominant vortices evolving near the dead zone's outer-edge. As long as the dead zone remains in the disc, the vortices can exist.

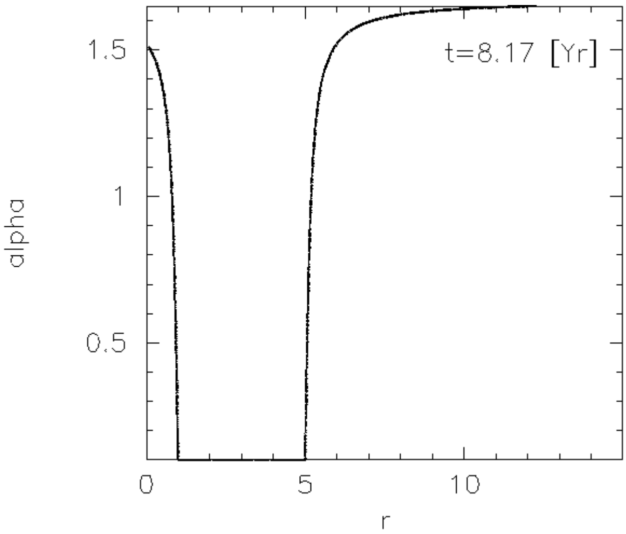



Fig. 1: Radial profile of the artificial viscosity $\alpha$ in our SPH simulation of the disc. $\alpha \in [0.1; 1.7]$. The radius is in AU.

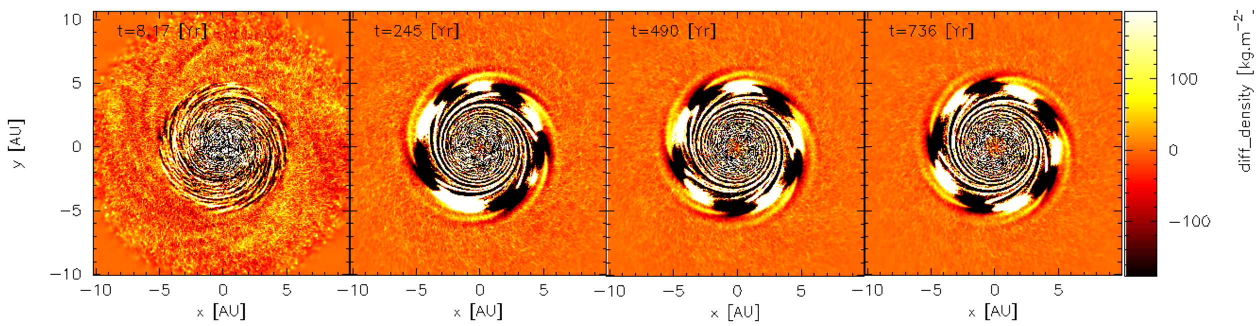



Fig. 2: Map of the density fluctuations near 5 AU. From left to right: 8.17, 245, 490 and 736 years.

### 4.2 *Time evolution of the density*

The radial surface density profile (Figure 3) shows the accretion of matter across the edges of the dead zone that allows to validate the $\mathcal{L}$-criterion.

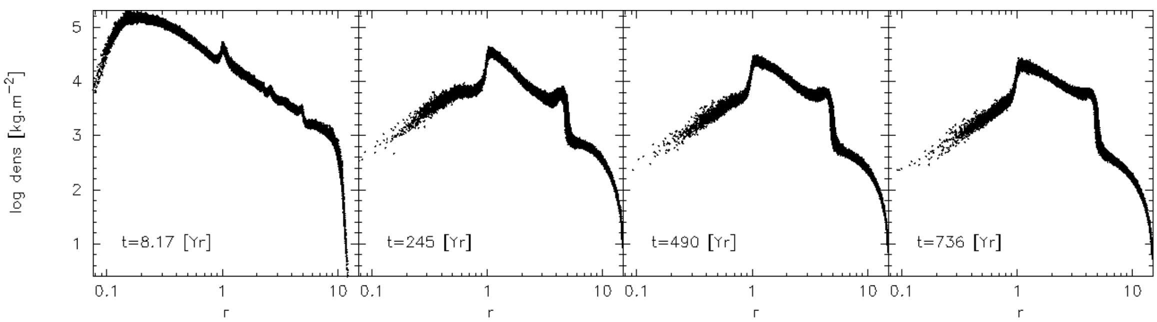


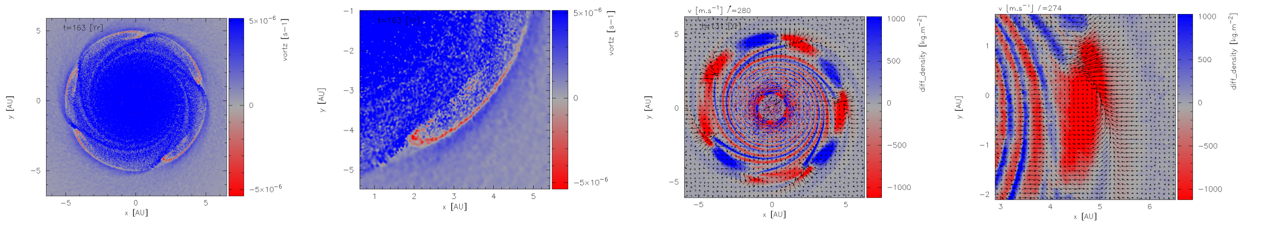

Fig. 3: Evolution of the radial density profile with time : 8.17, 245, 490 and 736 years. The radius $r$ is in AU.

### 4.3 Vorticity

Figure 4a and 4b show the vorticity ($\omega_z = (\nabla \times \mathbf{v})_z$) of the SPH particles in the disc. One sees five anticyclonic vortices coupled with five cyclonic vortices that appear less resolved (as can be seen in Figure 4b showing a close up of a vortex near 5 AU). The mode $m$=5 is dominant during all the simulation. No more merging of vortices is appreciable after the early evolution (Section 4.1).

### 4.4 Velocity field

In Figure 4c and 4d, the plot of the non-axisymmetric part of the gas velocity ($\mathbf{v}_{\rm gas} - \mathbf{v}_{\mathbf{K}}$, where $\mathbf{v}_{\mathbf{K}}$ is the Keplerian velocity) highlights the presence of cyclonic ($\Omega_{\rm vortices} \equiv +\Omega_{\rm K}$) and anticyclonic ($\Omega_{\rm vortices} \equiv -\Omega_{\rm K}$) vortices (near the outer edge of the DZ) and density waves. One clearly sees the rotation of the gas particles around underdensities (Figure 4d).



(a) Vorticity in the $z$ direction. (b) Vorticity: close up of a vortex at ≈5 AU. (c) Velocity field in the disc. (d) Velocity field: close up of a vortex near ≈5 AU.

Fig. 4: **Vorticity plot (a,b):** Blue: cyclonic (positive) vorticity and red: anticyclonic (negative) vorticity. **Velocity plot (c,d):** Colors represent the differential density, the non-axisymmetric part of the velocity field is superimposed.

## 5 Conclusions

The presence of a dead zone in the disc (Varnière & Tagger 2006) is a good tool to obtain an overdensity that allows the development of the Rossby Wave Instability. The presence of a viscosity in the disc, that is modeled by the use of the radial variation of the $\alpha$ artificial viscosity, is not an obstacle to the existence of the RWI. Velocity and vorticity plots clearly show the presence of Rossby Vortices that survive during a few thousand years. A mode $m$=5 is dominant during all the simulation. Other works show various evolution and merging of vortices from higher modes ($m$=4,6) to lower ones ($m$=3,2) (Varnière & Tagger 2006; Lyra et al. 2008, 2009). A comparison between Lagrangian (SPH) and Eulerian methods should be considered. Further work to investigate the growth of modes, the behaviour of dust in the 2D viscous disc in the presence of vortices and the growth of the RWI in a 3D viscous disc is in progress.

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## References

Barge, P. & Sommeria, J. 1995, A&A, 295, L1

Barrière-Fouchet, L., Gonzalez, J.-F., Murray, J. R., Humble, R. J., & Maddison, S. T. 2005, A&A, 443, 185

Gingold, R. A. & Monaghan, J. J. 1983, MNRAS, 204, 715

Güttler, C., Blum, J., Zsom, A., Ormel, C. W., & Dullemond, C. P. 2010, A&A, 513, A56+

Li, H., Finn, J. M., Lovelace, R. V. E., & Colgate, S. A. 2000, ApJ, 533, 1023

Lovelace, R. V. E. & Hohlfeld, R. G. 1978, ApJ, 221, 51

Lovelace, R. V. E., Li, H., Colgate, S. A., & Nelson, A. F. 1999, ApJ, 513, 805

Lyra, W., Johansen, A., Klahr, H., & Piskunov, N. 2008, A&A, 491, L41

Lyra, W., Johansen, A., Zsom, A., Klahr, H., & Piskunov, N. 2009, A&A, 497, 869
Meglicki, Z., Wickramasinghe, D., & Bicknell, G. V. 1993, MNRAS, 264, 691
Shakura, N. I. & Sunyaev, R. A. 1973, A&A, 24, 337
Tagger, M. 2001, A&A, 380, 750
Varnière, P. & Tagger, M. 2006, A&A, 446, L13
Weidenschilling, S. J. 1977, MNRAS, 180, 57
Zsom, A., Ormel, C. W., Güttler, C., Blum, J., & Dullemond, C. P. 2010, A&A, 513, A57+