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NEUTRINO TRANSPORT IN GRAVITATIONNAL SUPERNOVÆ SIMULATIONS : A SIMPLIFIED TREATMENT VIA A LEAKAGE SCHEME

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Abstract. We present a leakage scheme, a simplified model for the treatment of the neutrinos in core collapse supernovæ simulations. In the leakage scheme, the neutrinos are considered either fully at equilibrium with the fluid or streaming out freely. This approach is a quite rough approximation compared to a full transport scheme (Boltzmann solver), but it is much less CPU time consuming and hence is well suited, e.g., for parameter studies. This scheme has been written for CoCoNuT, a general relativistic hydrodynamics code which uses Godunov type methods.

Keywords: supernova, neutrino, numerical methods

1 Introduction

A core collapse supernova is the explosion of a massive star (roughly more than $8\mathrm{M}_\odot$). At the end of its life, a massive star has onion-like layers of nuclei, the most massive being at the center. Unless the star is very close to $8\mathrm{M}_\odot$ the core is composed of iron-like nuclei and the explosion is triggered by it reaching the Chandrasekhar mass ($\simeq 1.4\mathrm{M}_\odot$), when the degeneracy pressure of the electrons cannot balance the gravitation anymore.

The core collapses and looses degeneracy pressure as there are more and more electron captures with increasing density. Above $\rho \simeq 2.10^{14}$ g cm^{-3} strong interaction becomes repulsive. The collapsing core bounces, leaving a central compact object (proto-neutron star or black hole).

This bounce creates a pressure wave that becomes a shock propagating outwards. It looses a huge amount of energy photodissociating iron nuclei and soon stalls. Since we observe supernovæ, we know the shock has to revive. The ingredients implied to make the shock propagate again are still under investigation, and may involve neutrino heating, hydrodynamic instabilities (SASI, see Blondin et al. (2003), Foglizzo et al. (2007)), convection, magnetic field, general relativistic effects, ...

2 Leakage scheme

The leakage scheme is a way to avoid the computation of the neutrino distribution function, which would be done by solving the transport equation (this equation depends on time and 6 dimensions in phase space if no symmetry is assumed, and is often referred to as Boltzmann equation). Instead, the neutrinos are considered fully at equilibrium with the fluid (they enter the fluid equations) when trapped, and streaming freely when untrapped (they leave the simulation and remove some energy from the fluid). New fluid equations are implemented in the trapped regime, to satisfy the lepton number conservation. To create neutrinos three neutrino processes are considered : electron capture, pair production by electron positron annihilation and pair production by plasmon decay. Neutrino energies are averaged so that they only have a mean energy (grey scheme). Additional processes are considered to compute the neutrino mean free path : scattering off neutrons, scattering off nuclei, scattering off protons and absorption on neutrons (the latter is not relevant for neutrinos other than ν_e).

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2.1 *Neutrino processes*

2.1.1 Electron capture on free protons

Electron captures lower the electron density n_{e^-}. In CoCoNuT, we evolve the electron fraction Y_e

$$Y_e = \frac{n_{e^-} - n_{e^+}}{n_b} \tag{2.1}$$

The electron capture on free protons

$$p + e^- \rightarrow n + \nu_e \tag{2.2}$$

is treated as in Bruenn (1985) (eq. B12)

$$\frac{\partial Y_e}{\partial t} = \frac{1}{\rho}\frac{4\pi c}{8\pi^3(\hbar c)^3}\int_0^\infty w^2 dw \left(j(w)[1 - f_\nu(w)] - \frac{f_\nu(w)}{\lambda(w)} \right) \tag{2.3}$$

Where w is the neutrino energy, $f_\nu(w)$ is the distribution function of the neutrinos*, and $j(w)$ and $1/\lambda(w)$ are respectively the emissivity and absorptivity, defined by eqs. C13 and C15 of Bruenn (1985), with an extra $|V_{ud}|^2$ term. This CKM matrix term describes the mixing of mass eigenstates and gauge eigenstates of the quarks entering the hadronic matrix element (n; p).

Contrary to Bruenn (1985), we compute the integration every time for η_{np} and η_{pn}, the nucleon final state Pauli blocking, to avoid approximation problems.

$$\eta_{np} = \frac{2}{(2\pi\hbar c)^3}\int d^3p \; f_n(E)(1 - f_p(E)) \tag{2.4}$$

η_{pn} can be found by interchanging $n \leftrightarrow p$. $f_n(E)$ and $f_p(E)$ are Fermi-Dirac distribution functions with $E = p^2/2m$.

2.1.2 Electron capture on nuclei

The electron capture on nuclei

$$e^- + A \rightarrow A' + \nu_e \tag{2.5}$$

can be treated similarly, except that it requires the calculation of the corresponding emissivity $j(w)$ and absorptivity $1/\lambda(w)$, eq. C.27 and C.29 from Bruenn (1985).

2.1.3 Pair production

The pair production from electron positron annihilation

$$e^+ + e^- \rightarrow \nu_i + \bar{\nu}_i \qquad (i = e, \mu, \tau) \tag{2.6}$$

is implemented following Ruffert et al. (1996) (eq. B8-10 and B16).

The pair production from plasmon decay

$$\tilde{\gamma} \rightarrow \nu_i + \bar{\nu}_i \qquad (i = e, \mu, \tau) \tag{2.7}$$

is implemented following Ruffert et al. (1996) (eq. B11, B12 and B17).

2.2 *Neutrinosphere*

2.2.1 Effective chemical potentials

The neutrinosphere is computed as in Ruffert et al. (1996), with effective chemical potentials. $\mu_\nu = 0$ if the neutrinos are free streaming, $\mu_\nu = \mu_\nu^{eq} = \mu_e + \mu_p - \mu_n$ if the neutrinos are totally trapped. In the intermediate regime, the effective chemical potential is calculated as

$$\mu_\nu = \mu_\nu^{eq}(1 - \exp(-\tau)), \tag{2.8}$$

where τ is the optical depth defined in the next subsection. This is a self-consistent problem which we solve iteratively. The effective chemical potential is taken into account in the calculation of the electron capture and pair production rate, too.

*since we do not compute distribution functions in the leakage scheme, we have to assume that it is a Fermi-Dirac distribution.

2.2.2 Optical depth calculation

For each process considered (scattering off neutrons, scattering off nuclei, scattering off protons and absorption on neutrons), we compute a cross section, following Ruffert et al. (1996) and Rosswog & Liebendörfer (2003). The corresponding opacity is roughly proportionnal to the square of the neutrino energy.

This opacity (or inverse mean free path $1/\lambda$) is then integrated over a radial path to get the optical depth. With general relativistic corrections (within conformal flatness condition) this gives

$$\tau(r) = \int_r^\infty \frac{1}{\lambda} \frac{dr}{\frac{\alpha}{\phi^2} - \beta^r} \tag{2.9}$$

where α is the lapse, β^r is the radial part of the shift vector and ϕ id the conformal factor. The radius of the neutrinosphere R_ν is defined as $\tau(R_\nu) = 2/3$. The neutrinosphere is the limit which separates the two regimes (trapped at $r < R_\nu$, free streaming at $r > R_\nu$).

The neutrino escape time is then (from Ruffert et al. (1996))

$$t_{esc} = \frac{3(r - R_\nu)}{c} \tau(r) \tag{2.10}$$

2.3 Energy treatment

Neutrinos have a mean energy computed as

$$\langle \epsilon \rangle = T \frac{F_5(\eta_e)}{F_4(\eta_e)} \tag{2.11}$$

with T the fluid temperature, and F_5 and F_4 Fermi integrals defined by

$$F_k(\eta) = \int_0^\infty \frac{x^k dx}{1 + \exp(x - \eta)} \tag{2.12}$$

The energy removed is then $\langle \epsilon \rangle Y_\nu / t_{esc}$ if the neutrinos are trapped and $\langle \epsilon \rangle R$ if the neutrinos are free streaming (R is the total neutrino production rate).

In the regime where neutrinos are trapped, a neutrino pressure term that has to be taken into account, too (from Dimmelmeier et al. (2008)).

$$P_\nu = \frac{T^4}{6\pi^2 (\hbar c)^3} F_3(\eta_\nu) \tag{2.13}$$

3 Behavior during a simulation

3.1 Collapse and bounce

Starting with a progenitor from Heger and Woosley (see, e.g., Heger et al. (2005)), we run a simulation using CoCoNuT (see, e.g., Dimmelmeier et al. (2005)) with spherical symmetry, equation of state by Lattimer and Swesty (Lattimer & Swesty (1991)) and the leakage scheme. The collapse is induced by an initial velocity gradient.

At collapse, only electron captures are significant. The central density increases as free streaming neutrinos escape from the core. At a density of about 2.10^{12} g cm^{-3} neutrinos become trapped in the fluid. At about 5.10^{14} g cm^{-3}, the core bounces and the central density stabilises at 3.10^{14} g cm^{-3}, slightly above nuclear matter saturation density, creating the proto-neutron star. The central electron fraction at this time is 0.26 to 0.28, in good agreement with Liebendörfer (2005).

3.2 After bounce

The shock propagates outwards but looses a lot of energy by photodissociating heavy nuclei. It stalls at about 80 to 100km from the center. Pair production becomes non negligible because of the very high temperatures reached in this area (several MeV).

In these simulations the shock cannot revive because there is no neutrino heating and no convection or hydrodynamic instability (that would require more than one dimension).

The proto-neutron star contracts to compensate the energy loss by pair production and electron capture, such that the central temperature remains relatively constant. Together with the mass still falling onto the proto-neutron star, this slowly makes the shock move back to the center of the star (see Figure 1).

We then reach a point where the shock is at the edge of the proto-neutron star, which continues to contract as matter still falls on it. The only possible behavior is that the proto-neutron star collapses into a black hole. This is not yet visible on Figure 1 .

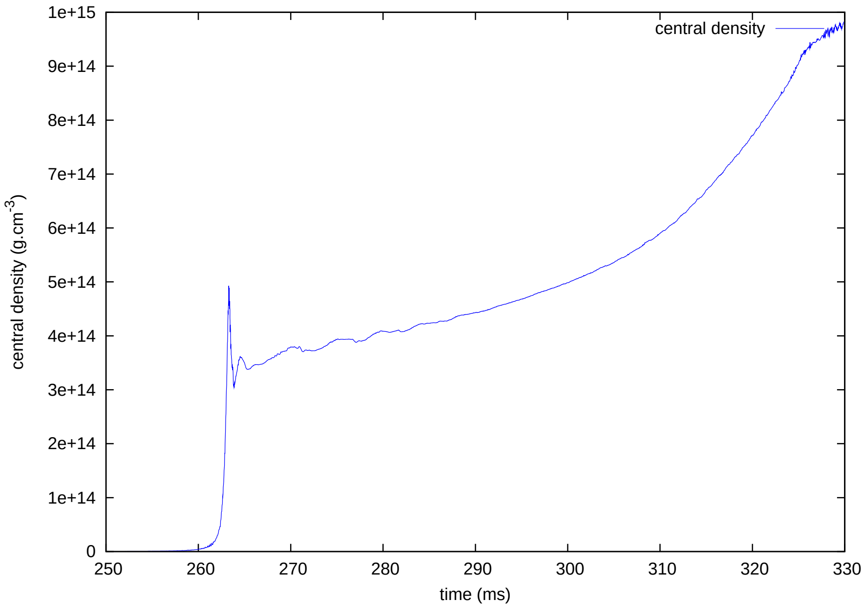

Fig. 1. Central density as a function of time

4 Conclusions

The leakage scheme is an approximation for the neutrino treatment in core collapse supernovæ. It is computationnally cheap compared to a Boltzmann solver but comes with some drawbacks. The leakage does not reproduce well the intermediate regime, where the neutrinos are not in equilibrium with the fluid but not yet free streaming either. This is the regime where the shock stalls, and consequently the leakage scheme implies too much deleptonization behind the shock.

Being aware of these approximations, the leakage scheme can be sufficient for some applications, in particular parameter studies where one cannot afford to use a Boltzmann solver.

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