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# LIBRATIONAL RESPONSE OF A THREE-LAYER TITAN

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**Abstract.** The knowledge of the rotational motion is an important piece of information about the interior structure and possible process acting inside the bodies. Recently, through the observations acquired by the space mission Cassini, the rotational motion of Titan has been determined. Here, we investigate the librational motion that are the variations around the uniform rotational motion. The librations present a wide spectrum of frequencies due to the orbital variations of the satellite. In this work, we focus on the librational signature of Titan in longitude by modeling the gravitational torque of Saturn with a non-keplerian orbit and internal couplings.

Two different timescales dominate the spectrum, long periods related to the motion of the nodes of the orbit and short periods related to the orbital period of the satellite. These long period librations have amplitudes almost independent of the distribution of mass and bring no information on the geophysical interior. On contrary, the short period librations are sensitive to the interior. For example the presence of an internal ocean increases strongly the amplitude of short period librations. However, it is necessary to take into account all librations (long and short) in order to interpret the spacecraft observations.

Keywords: Libration, Titan, structure, longitude, orbital analysis, frequency

## 1 Introduction

Stiles et al. (2008, 2010) have investigated the motion of landmarks at the surface of Titan from flybys of Cassini spacecraft. Titan's pole location and spin rate have been obtained by this method, and the presence of an internal ocean has been suggested (Lorenz et al. 2008). They have determined an obliquity of about 0.3 deg and that the spin rate differs slightly from the synchronous spin-orbit resonance. Bills & Nimmo (2008) have shown that this obliquity is not consistent with a solid Titan in Cassini State and they suggested the existence of an internal liquid ocean to solve this problem.

The existence of the liquid ocean under Titan's surface has been studied with several methods. The Permittivity, Wave and Altimetry (PWA) instrument has measured the electric field during the *Huygens* descent through the atmosphere of Titan. These measurements revealed the possibility of an ice/ocean interface at 30-60 km depth (Béghin et al. 2010). The actual determined obliquity combined with the moment of inertia of Titan (Iess et al. 2010) and studies of Bills & Nimmo (2011) or Baland et al. (2011) suggest the existence of an internal liquid ocean as decoupling mechanism between the shell and the solid interior. A more recent determination of the variation of the quadrupole field (Iess et al. 2012) and Love number $k_2$ are consistent with the presence of a liquid subsurface ocean.

Here we investigate the librational motion of Titan as another method of deep structure analysis. This method has been developped and used for the Galilean satellites (Rambaux et al. 2011; Baland & Van Hoolst 2010) and for Titan for an elliptic motion (Van Hoolst et al. 2009). Librations in longitude are departures from the uniform rotational motion, caused by the variations of orbital velocity induced by the eccentricity of Titan's orbit. Here, we investigate the librational motion of Titan by including the perturbed orbital motion.

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## 2 Orbital and librational motion of Titan

### 2.1 Longitudinal Libration equations in three layer model

We assume that Titan is composed of three principal layers: a solid ice shell, a liquid ocean and a solid inner core. The longitudinal libration equations are given by coordinate transformations of the angular momentum equation for a given layer $i$

$$\frac{d\boldsymbol{H}_i}{dt} = \boldsymbol{\Gamma}_i, \tag{2.1}$$

where $\boldsymbol{H}_i$ is the angular momentum expressed as $\boldsymbol{H}_i = [I]_i\boldsymbol{\omega}_i$ with $[I]_i$ the tensor of inertia, $\boldsymbol{\omega}_i$ the rotation vector and $\boldsymbol{\Gamma}_i$ the sum of the internal and external gravitational and pressure torques. Following Van Hoolst et al. (2009), Rambaux et al. (2011) derived from the third component of the angular momentum equation an expression of the equations of longitudinal libration:

$$C_s\ddot{\gamma}_s + 3n^2[(B_s - A_s) + (B'_s - A'_s)] + 2K_{int}\gamma_s - 2K_{int}\gamma_i = 3n^2[(B_s - A_s) + (B'_s - A'_s)](\nu - M - \theta_{0s}) \tag{2.2}$$

$$C_i\ddot{\gamma}_i + 3n^2[(B_i - A_i) - (B'_i - A'_i)] + 2K_{int}\gamma_i - 2K_{int}\gamma_s = 3n^2[(B_i - A_i) - (B'_i - A'_i)](\nu - M - \theta_{0i}) \tag{2.3}$$

where indices are $s$ for the shell and $i$ for the inner core, with $A_l$, $B_l$ and $C_l$ the principal moment of inertia of layer $l$ ($A_l < B_l < C_l$, index $'$ denotes the moment of inertia difference caused by the pressure of the ocean), $\gamma_l$ the librational angle defined as $\gamma = \theta - M - \theta_0$ with $M$ the mean anomaly, $\theta$ the rotation angle and $\theta_0$ the initial value of $\theta$, $n$ the mean motion of Titan, $\nu$ the true longitude of Titan and $K_{int}$ the amplitude of the internal gravitational torque between the shell and the interior.

For a purely keplerian motion, the difference $\nu - M$ can be developed as a series of mean anomaly $M$ and eccentricity $e$. Here we consider also the perturbations from other satellites and the Sun, so we expect frequencies corresponding to the perturbing bodies.

### 2.2 Frequency analysis method

To investigate the orbital motion of Titan, we perform a frequency analysis of the true longitude of Titan obtained from the JPL's Horizons ephemerides (Giorgini et al. 1996) for 400 years time span. We used the frequency analysis method developped by Laskar (1988, 2003) to identify the frequencies present in the true longitude. This method is implemented in the TRIP software developped by Gastineau & Laskar (2012). Table 1 gives the frequencies of the different components of the true longitude, where $L_6$ is the linear part of the mean longitude of Titan, $\Omega$ is the longitude of the node of Titan or Iapetus ($\Omega_6$ or $\Omega_8$), $\varpi$ is the longitude of the pericenter of Titan or Iapetus ($\varpi_6$ or $\varpi_8$) and $L_s$ the mean longitude of the Sun.

The most important term in the analytical decomposition of the true longitude corresponds to the orbital period of Titan with an amplitude of two times the eccentricity ($e = 0.028$). We also recognize the perturbation terms from the Sun by the saturnian annual (29.44 years) and semi-annual (14.72 years) excitation of Titan's true longitude ($L_s$ and $2L_s$ terms respectively). We can then include these decomposition of the true longitude in the equations of libration to obtain the longitudinal libration of Titan for a chosen internal structure model.

### 2.3 Internal structure models

Equations (2.2) and (2.3) show that the librations are dependent on the internal structure of the satellite through the moments of inertia. Recent moments of inertia determination by Iess et al. (2010) has allowed to get information on Titan's internal structure. Fortes (2012) has explored the chemical and thermal parameters consistent with the moment of inertia of Titan, and constructed a range of possible internal structures. In our analysis, three different internal structure models have been selected from Fortes (2012). One of them includes internal liquid ocean while the other ones are solid models with different interior compositions and densities. Even if the internal liquid ocean seems to be confirmed as the analyses progress, we keep the solid models for results comparison. The different interior structures used here are described by Fortes (2012).

## 3 Results and discussion

For each internal structure model and each frequency of the true longitude, we compute the libration angle solution of the shell as $\gamma_s = \sum_j A_j \sin(\omega_j t + \phi_j)$, where $A_j$ are the amplitudes of librations, $\omega_j$ the frequencies and $\phi_j$ the phases. The librational motion of Titan's equator in the light internal ocean model (model 1) is plotted on Figure 1. The motion is dominated by long period terms from the annual and semi-annual components (29.44 and 14.72 years). The short period terms are visible at a lower scale in the thickness of the curve with smaller amplitudes.

**Table 1.** Librations in longitude of Titan due to orbital forcing (deviation in meters). The orbit comes from the Horizons ephemerides taken over 400 years (Giorgini et al. 1996). Initial date is J2000. The model 1 is the light internal ocean model (ocean density of 1000 kg m$^{-3}$), the model 2 is a solid model of pure water ice and a rocky core, and the model 3 is a solid model of pure water ice including a liquid iron core (developed by Fortes 2012).

| | | **Amplitude of libration [m]** | | | |
|---|---|---|---|---|---|
| **Frequency [rad/day]** | **Period [day]** | **Model 1 (ocean)** | **Model 2 (solid)** | **Model 3 (liquid core)** | **Identification** |
| 0.39402 | 15.946 | -319.036 | -52.032 | -52.034 | $L_6 - \varpi_6$ |
| 0.78803 | 7.973 | -1.422 | -0.232 | -0.232 | $2L_6 - 2\varpi_6$ |
| 0.39408 | 15.944 | -1.520 | -0.248 | -0.248 | $L_6 - 2\varpi_8 + 2\Omega_6$ |
| 0.00117 | 5376.633 | 552.772 | 560.119 | 560.118 | $2L_s$ |
| 0.00058 | 10750.365 | 470.287 | 471.838 | 471.838 | $L_s$ |
| 0.39290 | 15.992 | -0.851 | -0.139 | -0.139 | $L_6 + \Omega_6 - 2L_s$ |

Table 1 gives the decomposition of the libration angle of the shell for each orbit excitation frequency in the case of a light internal ocean model (lowest density of water, model 1), and pure water ice models with and without liquid iron core (model 3 and 2 respectively). As expected, the most important terms are the annual and semi-annual components with amplitudes for model 1 of 470.287 and 552.772 meters, respectively. However, models 2 and 3 (solid) have both amplitudes of 471.8 and 560.1 meters, respectively. The differences of amplitude for these small frequencies are just a few meters, mostly due to the oceanic pressure on the solid surfaces. The low frequencies librations are almost independent of the structure models. The gravitational torque dominates the inertia of the body for low frequencies, so the amplitudes of librations are almost equal to the magnitudes of perturbations.

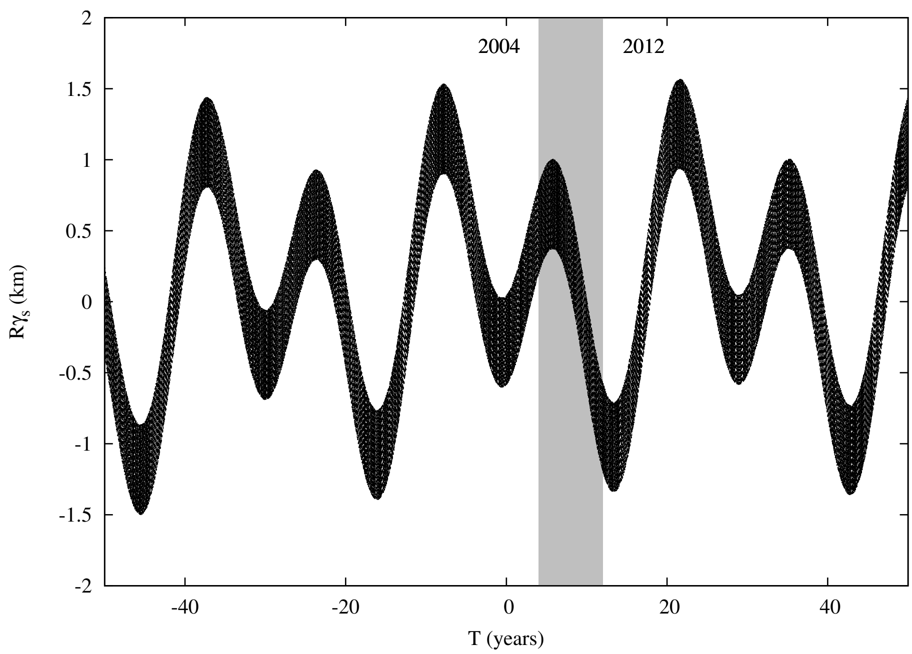


**Fig. 1.** Time evolution of the libration in longitude angle of Titan over 100 years. The origin of time is J2000. The grey band represents the arrival date of Cassini in Saturn system and the today value.

At high frequencies (first terms of Table 1), the amplitudes of libration are discriminating for the different models. The internal ocean model has a deviation of 319.036 meters at the orbital frequency while the solid models have about 50 meters deviations. Indeed, for this range of frequencies, the inertia of Titan is significant on its behavior and dominates the gravitational torque.

In addition, we note that, the forcing frequencies are far from the proper frequencies of the models ($\omega_1 = 2.147\ 10^{-2}$ rad d$^{-1}$ and $\omega_2 = 6.502\ 10^{-3}$ rad d$^{-1}$ for model 1 and $\omega_s = 7.372\ 10^{-3}$ rad d$^{-1}$ for model 2 and model 3) so no resonance can occur. Taking into account the atmospheric torque in the libration angle solution as in Van Hoolst et al. (2009), we had $\Gamma(t) = \Gamma_A \sin(\omega_A t + \phi_A)$ to the torques applied on Titan, with $\Gamma_A = 1.6\ 10^{17}$ Nm (Tokano & Neubauer 2005)(the amplitude of the gravitational torque is 2.63 $10^{20}$ Nm for model 1). This forcing term has a frequency $\omega_A$ equals to the Saturn's semi-annual frequency (period of 14.72 years). At this frequency, the libration amplitude for the model 1 reaches 791.555 meters i.e. 238.783 meters of atmospheric contribution.

## 4 Conclusions

An accuracy of 100 meters on Titan's equator deviation would be useful to detect the presence of the internal ocean with the librations. For the moment, the SAR landmarks positions error is at best 850 meters (Stiles et al. 2008). More data are needed to obtain a better constraint on the rotation model and landmarks locations. During the actual Cassini mission lifetime, Titan must have undergo deviations of about one kilometer. This deviation should be sufficiently large to be detected by the SAR method and included in the rotation model of Titan.

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## References

Baland, R.-M. & Van Hoolst, T. 2010, Icarus, 209, 651

Baland, R.-M., Van Hoolst, T., Yseboodt, M., & Karatekin, Ö. 2011, A&A, 530, A141

Béghin, C., Sotin, C., & Hamelin, M. 2010, Comptes Rendus Geoscience, 342, 425

Bills, B. G. & Nimmo, F. 2008, Icarus, 196, 293

Bills, B. G. & Nimmo, F. 2011, Icarus, 214, 351

Fortes, A. D. 2012, Planet. Space Sci., 60, 10

Gastineau, M. & Laskar, J. 2012, TRIP 1.2.0, TRIP Reference manual, IMCCE, Paris Observatory, `http://www.imcce.fr/trip`

Giorgini, J. D., Yeomans, D. K., Chamberlin, A. B., et al. 1996, in Bulletin of the American Astronomical Society, Vol. 28, AAS/Division for Planetary Sciences Meeting Abstracts #28, 1158

Iess, L., Jacobson, R. A., Ducci, M., et al. 2012, Science, 337, 457

Iess, L., Rappaport, N. J., Jacobson, R. A., et al. 2010, Science, 327, 1367

Laskar, J. 1988, A&A, 198, 341

Laskar, J. 2003, arXiv:math/0305364

Lorenz, R. D., Stiles, B. W., Kirk, R. L., et al. 2008, Science, 319, 1649

Rambaux, N., Van Hoolst, T., & Karatekin, Ö. 2011, A&A, 527, A118

Stiles, B. W., Kirk, R. L., Lorenz, R. D., et al. 2008, AJ, 135, 1669

Stiles, B. W., Kirk, R. L., Lorenz, R. D., et al. 2010, AJ, 139, 311

Tokano, T. & Neubauer, F. M. 2005, Geophys. Res. Lett., 32, 24203

Van Hoolst, T., Rambaux, N., Karatekin, Ö., & Baland, R.-M. 2009, Icarus, 200, 256