# SIGNATURES OF ROTATION IN OSCILLATION SPECTRA

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**Abstract.** Rotation makes oscillation spectra of rapidly rotating stars much more complicated. Hence, new strategies need to be developed in order to interpret such spectra. In what follows, we describe how multi-colour photometric mode visibilities can be generalised to rapidly rotating stars, while fully taking into account centrifugal deformation and gravity darkening. We then go on to describe some first results as well as a strategy for constraining mode identification.

Keywords: Asteroseismology, Stars: oscillations, Stars: rotation, multi-colour photometry, mode identification

#### 1 Introduction

The space missions CoRoT (Baglin et al. 2009; Auvergne et al. 2009) and Kepler (Borucki et al. 2009) are revealing very rich pulsational spectra in rapidly rotating stars. For instance, the  $\delta$  Scuti star HD 181555, a primary target of the CoRoT mission, may have up to 2000 individual pulsation modes (Michel, private communication). It is becoming increasingly clear that in order to identify modes, *i.e.* find the correspondence between observed frequencies and theoretically calculated modes, one will need more sophisticated methods than simply using equidistant patterns such as what is done in Echelle diagrams. One such approach, commonly used in non-rotating stars, is multi-colour photometry. This approach consists in observing pulsation modes in multiple photometric bands, calculating the amplitudes ratios and/or phase differences between the different bands, and comparing these with theoretical predictions. In what follows, we describe how this approach can be generalised to rapidly rotating stars.

## 2 Formalism

Previous efforts to calculate mode visibilities in rapidly rotating stars either approximated the effects of rotation on the pulsation modes (e.g. Daszynska-Daszkiewicz et al. 2007) or were restricted to disk-integration factors (e.g. Lignières & Georgeot 2009). Nonetheless, these studies established various important results, such as the fact that amplitude ratios depend on m, the azimuthal order, and i, the inclination, in rotating stars (Townsend 2003; Daszynska-Daszkiewicz et al. 2007) and that modes with intermediate  $\ell - |m|$  values tend to become more visible as the rotation rate increases, since these become chaotic modes with irregular node spacing (Lignières & Georgeot 2009). In order to go further, we combine a complete 2D treatment of the effects of rotation with realistic calculations of mode visibilities.

Luminosity variations in a pulsating star,  $\Delta E(t)$ , are given by the following equation:

$$\Delta E(t) = \Re \left\{ \iint_{\text{Vis.Surf.}} \delta I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot \vec{dS} + \iint_{\text{Vis.Surf.}} I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs.}} \cdot \delta(\vec{dS}) \right\},$$
(2.1)

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where  $\delta$  represents a Lagrangian perturbation,  $\Re\{\dots\}$  the real part of some quantity, "Vis.Surf." the visible surface, dS a surface element,  $\vec{e}_{obs.}$  a unit vector towards the observer, and I the specific intensity for a given orientation,  $\mu = \vec{e}_{obs.} \cdot dS / \| dS \|$ , effective gravity,  $g_{eff}$ , and temperature,  $T_{eff}$ . In what follows, we will assume that the specific intensity has been multiplied by the transmission curve of the instrument and/or filter, and has been integrated over the wavelength spectrum. Variations of the boundary between the visible and hidden side of the star, caused by the pulsations, are neglected since these are of second order in terms of the Lagrangian displacement. In order to apply Eq. (2.1) to pulsation modes in rapidly rotating stars, we adapted each of the intervening terms to the structure of a centrifugally distorted star. In particular, a grid of Kurucz atmospheres was calculated so as to have the correct intensities (and associated derivatives) at each latitude, thereby taking gravity (and limb) darkening into account. For more details, we refer the reader to (Reese et al., submitted).

### 3 Results

**Overall visibilities** As a first step, we deal with mode visibilities in a single photometric band. The intrinsic mode amplitudes are normalised so that the maximal displacement, multiplied by the square of the co-rotating frequency,  $(\omega + m\Omega)^2$ , is constant. Figure 1 displays visibilities in CoRoT's photometric band at four different inclinations, for 2  $M_{\odot}$  models based on the Self-Consistent Field (SCF) method (Jackson et al. 2005; MacGregor et al. 2007), at two different rotation rates. The different colours indicate the values of  $\ell$  carried over from the non-rotating case. In agreement with Lignières & Georgeot (2009), we find that modes with intermediate  $\ell - |m|$  values become more visible as the rotation rate increases and at higher inclinations. However, even when the star is equator-on, some of the island modes, *i.e.* the rotating counterparts to modes with low  $\ell - |m|$  values, still remain more visible, which is different from what was found previously. The pole-on configuration is particularly interesting because the non-axisymmetric modes cancel out, thereby leaving a relatively regular frequency spectrum in which island modes stand out (see also Lignières & Georgeot 2009). This suggests that rapidly rotating pole-on stars might be a good starting point for asteroseismic investigations and has motivated recent searches for pulsations in Vega (Böhm et al. 2012).



Fig. 1. Mode visibilities in CoRoT's photometric band, for two different rotation rates. Colours indicate the  $\ell$  values.

Amplitude ratios We now turn our attention to amplitude ratios. Figure 2 displays various amplitude ratios in the Geneva photometric system for different configurations and rotation rates. As can be seen in the first column, although modes with different azimuthal orders have the same amplitude ratios in non-rotating stars, this no longer applies to rotating stars, as was previously found in Townsend (2003); Daszynska-Daszkiewicz et al. (2007). The same applies to inclination but to a lesser extent, since amplitude ratios of a given mode still remain similar even at rapid rotation rates, as shown in the second column. The third column illustrates what happens if modes with the same  $(\ell, m)$  values and inclination are selected. The amplitude ratios can be quite similar even at rapid rotation rates. Such similarities are expected based on the asymptotic calculations by Pasek et al. (2012) which predict a similar lateral structure for these modes. Nonetheless, the middle panel at  $\Omega = 0.4\Omega_{\rm C}$  shows that differences can arise between such amplitude ratios – these can be caused by avoided crossings, departures from the asymptotic regimes, or competing contributions to the mode visibilities.



**Fig. 2.** Amplitude ratios. Each row corresponds to a different rotation rate, and each column to a different configuration. The values connected by dotted lines correspond to the parameter in the upper line of each title.

### 4 Constraining mode identification

It is then interesting to investigate whether it is possible to constrain mode identification from multi-colour photometry. In Fig. 3, we randomly choose an island mode in a model rotating at  $\Omega = 0.6\Omega_{\rm C}$  with an inclination of  $i = 60^{\circ}$ , and find 9 other modes out of a list of over 3000 modes, with the most similar amplitude ratios. The upper left plot displays the 10 sets of amplitude ratios. The upper middle panel shows the frequencies and the rank, which measures the degree of resemblance, 10 corresponding to the original mode. The upper right panel shows the most recurrent frequency differences convolved by a Gaussian. The vertical dotted and dashed lines display twice the rotation frequency, the large separation and half the large separation. Finally, the second and third rows show the meridional cross-sections of the 10 modes. The  $(n, \ell, m)$  values, carried over from the non-rotating case, are indicated below each cross-section. As can be seen, all of the selected modes are island modes, most of which have the same  $\ell$  and |m| values. Accordingly, the frequencies follow a specific pattern as confirmed by the upper middle and right panels. If we had selected, say, an m = 2 island mode, another frequency pattern would have emerged with a recurrent  $4\Omega$  spacing. Hence, by studying the resultant frequency pattern, one can hope to constrain the azimuthal order and possibly the harmonic degree. Furthermore, this yields the rotation rate, as well as the large frequency separation.

#### 5 Discussion

The next step in this work will be to include non-adiabatic effects. Indeed, this will allow us to obtain realistic effective temperature variations at the stellar surface, which is crucial for obtaining accurate mode visibilities (e.g. Dupret et al. 2002). Such calculations require thermally balanced rotating models, such as what is currently being produced in the ESTER project (Rieutord & Espinosa Lara 2009; Espinosa Lara 2010). Nonetheless, the current results may still provide a good qualitative description of the effects of rotation on multi-colour mode visibilities. In particular, rapidly rotating stars seen pole-on may be promising asteroseismic targets, given their simplified and more regular pulsation spectra. Furthermore, the above strategy for constraining mode identification does not require a full knowledge of the theoretical predictions and could already be applied to stars for which a sufficient number of modes have been observed in multi-colour photometry.

Once pulsation modes are identified in rapidly rotating stars, then it may be possible to apply inversion methods which will allow us to probe the rotation profile and stellar structure. This would provide valuable constraints on internal mixing and transport processes, and enable better predictions of the evolution and



Fig. 3. In this plot, an island mode was randomly chosen, and 9 other modes with the most similar amplitude ratios were selected. See text for details.

chemical yields of rapidly rotating stars.

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