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LAYERED DOUBLE DIFFUSIVE CONVECTION: FROM EARTH OCEANS TO GIANT PLANET INTERIORS.

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Abstract. Many unknowns remain concerning the internal structure and composition of giant gaseous planets. The existence and the properties of an hypothetical central core, in particular, are still debated.

Contrary to conventional interior models for giant (exo)planets, we consider an inhomogeneous mixing of heavy elements in the gaseous H/He envelope of these objects. As in the oceans, such compositional gradients can give rise to layered convection which impedes large scale convection, yielding a hotter, super adiabatic interior. As a result, the metal enrichment predicted by this model is up to 30 to 60% larger than previously thought for Jupiter and Saturn. However, metals are preferentially redistributed in the gaseous envelope and coreless models can be found for Jupiter.

This inefficient, layered convection, yielding a slower cooling, can help to explain anomalously inflated Hot Jupiters, but also opens a new window on our understanding of giant planet formation and history inside our Solar System.

Keywords: Planets and satellites: interiors, Convection

1 The double diffusive instability

In hydrodynamic experiments, it is well known that the presence of a compositional gradient, causing a gradient of mean molecular weight, can change the properties of the mean flow that develops in a stratified fluid. This is due to the fact that, as temperature, the mean molecular weight influences the buoyancy of the fluid and can (de)stabilize the whole medium against the convective instability (Schwarzschild & Härm 1958; Ledoux 1947); i.e. the instability which drives a denser material to sink and a lighter one to rise until a stable stratification is reached.

However, because diffusive processes are also at play, even a "gravitationally stable" stratification of solute and temperature can be unstable. Roughly, this "double diffusive instability", first theorized by Stern (1960), can arise if the diffusivity of one of the quantities (in general heat) is significantly greater than the other. Two cases are of particular interest. The first one, known as the *fingering case*, can arise when temperature plays a stabilizing role[i] and the mean molecular weight is destabilizing. A perfect example is given by the well known "salt fingers" that form when warm salty water is put on top of cold fresh water. Then, because of the higher thermal diffusivity, a small eddy of salty water that is displaced downward will cool efficiently and the eddy will continue to sink as the positive buoyancy due to the higher temperature will not be able to counteract the negative buoyancy due to the higher mean molecular weight.

The second is the *double diffusive case.* It arises when the medium is hotter downward and has a mean molecular weight which increases with depth. Then, a hot, dense, rising eddy still has a negative buoyancy pushing it back downward, but, as it looses heat, it will have a negative buoyancy when arriving at its starting point. An oscillating motion will thus develop in the fluid, much like gravity waves, but whose amplitude will grow with time (Stern 1960; Stevenson 1979; Rosenblum et al. 2011).

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[i]Temperature has a stabilizing effect if it increases upward or decreases less steeply than the adiabatic temperature gradient of the fluid considered. In the other case, temperature acts to destabilize the fluid and create convective motion. On the contrary, mean molecular weight plays stabilizing role by decreasing upward.

2 Transport properties of double diffusive convection

Both salt fingers and double diffusive convection are seen in experiments and in the oceans where their peculiar transport properties have a strong influence on the thermohaline circulation (Stern 1960; Turner & Stommel 1964; Schmitt 1994). And this is precisely because of these very peculiar heat transport and mixing properties that fluids developing double diffusive convection are of interest in an astrophysical context. Indeed, when modeling the evolution of stellar and substellar objects, quantifying the ability of the material to transport energy and to mix chemical elements is of prime importance. It will directly determine the rate of evolution of the body. As double diffusive convection is thought to be present in moderately massive stars (Schwarzschild & Härm 1958; Merryfield 1995), white dwarfs, and giant planets (Stevenson & Salpeter 1977; Stevenson 1979, 1985; Chabrier & Baraffe 2007; Leconte & Chabrier 2012) studying its macroscopic properties is thus mandatory to gain accuracy in the modeling of the aforementioned objects.

To model double diffusive convection[ii], the picture depicted in the previous section is incomplete as it only relies on linear stability analysis, and does not tell us what the mean flow is going to look like when the instability is fully developed. Indeed, both more sophisticated analytical models and numerical experiments show that depending on the ratio of the thermal and molecular diffusivities (κ_T and κ_μ), of the viscosity (ν), and of the sign and relative strengths of the imposed large scale gradients of buoyancy due to both temperature and mean molecular weight inhomogeneities, the mean flow can present different behaviors (Radko 2003; Rosenblum et al. 2011; Mirouh et al. 2012).

In the double diffusive case, the dimensionless parameter governing the selected behavior is the inverse density ratio,

$$R_\rho^{-1} \equiv \frac{\alpha_\mu}{\alpha_T} \frac{\left.\frac{\partial \ln \mu}{\partial \ln P}\right|_T}{\left.\frac{\partial \ln T}{\partial \ln P}\right|_\mu - \left.\frac{\partial \ln T}{\partial \ln P}\right|_{\mu,\mathrm{ad}}} \equiv \frac{\alpha_\mu}{\alpha_T} \frac{\nabla_\mu}{\nabla_T - \nabla_{\mathrm{ad}}}, \tag{2.1}$$

where $\alpha_T \equiv -\left.\frac{\partial \ln \rho}{\partial \ln T}\right|_{P,\mu}$, $\alpha_\mu \equiv \left.\frac{\partial \ln \rho}{\partial \ln \mu}\right|_{P,T}$, (Stern 1960; Rosenblum et al. 2011). At low forcing, i.e. at high R_ρ^{-1}, the medium is stable against the double diffusive instability and the fluid remains stably stratified. For $R_\rho^{-1} < (1+Pr)/(\tau+Pr)$, where $Pr = \nu/\kappa_T$ is the usual Prandlt number and $\tau = \kappa_\mu/\kappa_T$ is the inverse Lewis number, the instability grows. For $R_\rho^{-1} < 1$, which is the usual *Ledoux* criterion for convective instability, large scale convective motion develops independently of the diffusivities.

In the double diffusive regime, the finite amplitude flow depends on a critical inverse density ratio $R_{\min}$ which corresponds to the point where the solute to heat buoyancy flux ratio ($\equiv \gamma^{-1}$) stops increasing when R_ρ^{-1} decreases. $R_{\min}$ depends on the characteristics of the medium and can be estimated experimentally or numerically (Radko 2003; Rosenblum et al. 2011; Mirouh et al. 2012). For $R_\rho^{-1} > R_{\min}$, the forcing is sufficient to trigger the instability and create small scale turbulence (also called homogenous double diffusive convection) but the amplitude of the latter remains small. Transport properties in this state are more adequately represented by an eddy diffusion formalism, and the energy flux that can be transported is very limited (Rosenblum et al. 2011).

On the contrary, when $1 < R_\rho^{-1} < R_{\min}$, the fluid does not remain long in this state of homogenous double diffusive convection. As the instability grows, the system quickly develops thermo compositional layers of well mixed, convective regions separated by thin diffusive interfaces where both temperature and mean molecular weight undergo a sudden jump (Radko 2003; Rosenblum et al. 2011). In this regime, called the *layered convection* regime, the transport is much more efficient than in the case described above but is still fairly inefficient compared to large scale convection. Interestingly, Rosenblum et al. (2011) have shown that, as for large scale convection, the heat transport efficiency in the well mixed convective layers (measured by the Nusselt number which is the ratio of flux transported by convection over the flux transported by diffusion) roughly scales as a power law of the Rayleigh number which quantifies the thermal forcing (see Leconte & Chabrier (2012) for details).

3 Implications for giant planet structure

Thus to study the possible impact of layered convection on the structure of giant planets, we have developed an analytical parametrization as follows. We consider that a layered zone consists of a large number of well

[ii] The fingering case is not detailed here as it is of lower relevance in the astrophysical context. For details, the reader is referred to Stellmach et al. (2011) and reference therein.

mixed convectively unstable layers, separated by thin diffusive interfaces within which the large stabilizing compositional gradient completely inhibits convective motions. In the convective layers, the relationship between the flux to be transported and the thermal gradient is determined by the usual mixing length theory (Hansen & Kawaler 1994) but where the mixing length, $\alpha = l/H_P$, is assumed to be equal to the layer size (l; assumed small compared to the planet radius). H_P is the pressure scale height. The mixing length theory is also extended so that various power law exponents can be used. In the diffusive interfaces, the thermal gradient is equal to the one needed to transport all the flow by diffusive processes (thermal conduction by the free electrons in giant planets case).

As described in Leconte & Chabrier (2012), using this parametrization of layered convection, we were able to build internal structure models of Jupiter and Saturn with a *continuous* gradient of heavy elements in the gaseous envelope. These models fulfill all the available observational mechanical constraints, i.e. the mass, radius, rotation rate, gravitational moments (J_2 and J_4), and atmospheric heavy elements abundances derived from in situ measurements.

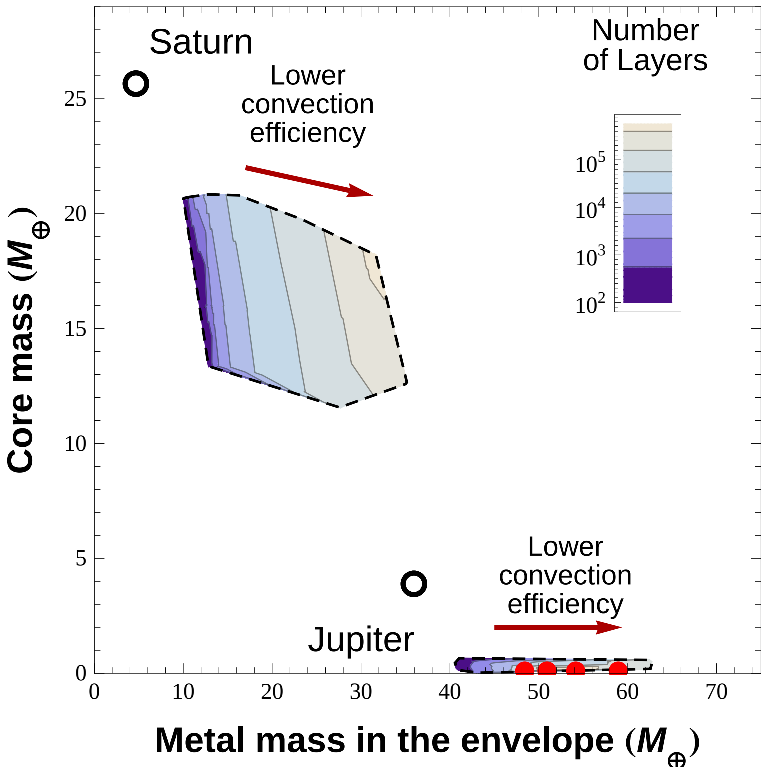

Fig. 1. Mass range of heavy elements in the core and in the envelope consistent with all observational mechanical constraints, for different numbers of layers, for Jupiter (bottom right) and Saturn (upper left). The open dots at the upper left of each region correspond to the homogeneous interior models. As the number of semiconvective layers increases, the efficiency of convection decreases, and the heavy element mass fraction increases to counteract the radius increase induced by the planet's higher internal temperature. The metals initially present in the core are then redistributed within the envelope. For Jupiter, solutions with no core at all can be found for the non adiabatic models (red dots).

As shown in Fig. 1, distribution of the heavy elements in the planet differs significantly between models with a well mixed envelope (homogeneous, adiabatic reference models) and inhomogeneous models where layered convection is present. There are two main differences:

- when a heavy element gradient is allowed in the envelope, models with a less massive core are preferred. The heavy elements are redistributed within the gaseous envelope which is more metal rich. This effect is quite independent (to zeroth order) of the change imposed to the thermal structure by the layered convection and just stems from the fact that, when the "envelope homogeneity" constraint is relaxed, measured values of J_2 and J_4 favor structures with smaller cores. This is why models with layered convection do not tend continuously towards the reference models when the number of convective/diffusive cells is decreased.

In the case of Jupiter, because of its less massive core, this redistribution of the heavy elements has a particularly important effect as models without any core can be found to match the observational constraints.

- when the number convective/diffusive cells is increased, i.e. when the size of each individual convective layer (l) is decreased, the superadiabaticity and temperature gradient in the planet is increased. This stems from the fact that layered convection is less and less efficient and a steeper gradient is needed to transport the same heat flux. The mean internal temperature is thus higher. To keep the same observed planetary radius, a higher metal enrichment of the envelope is needed to counteract the decrease in density caused by this higher internal temperature. A decrease in the convective layer size thus entails an increase of the total heavy element mass in the object, as seen in Fig. 1 and detailed in Leconte & Chabrier (2012).

4 Concluding remarks

These points have strong implications concerning the present composition of Solar System giant planets, and their formation history. Indeed, if a significant superadiabaticity is present in Jupiter and Saturn, their total metal mass fraction could be up to 30-60% higher than previously estimated on the basis of adiabatic, homogeneous models. This higher enrichment could completely change our vision of the heavy element budget during giant planets formation. It suggests a higher density of solids in the nebula and an early and efficient capture of planetesimals for ours and, probably also, extrasolar giant planets, helping to solve the traditional formation timescale problem of giant planets (Pollack et al. 1996).

However, the possibility of less massive cores inferred here revives the question of the origin of the heavy elements in the gaseous envelope of giant planets. Indeed, the possibility that Jupiter was formed with a very small core (less massive than $5\,M_\oplus$), if any, as inferred here, is difficult to conciliate with the very efficient accretion suggested above. To address this issue, we must thus understand to what extent the enrichment of the gaseous envelope of giant planets can be primordial or caused by the erosion and dissolution of a massive initial core.

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