SF2A 2013
L. Cambrésy, F. Martins, E. Nuss and A. Palacios (eds)

IMPACT OF TIDAL INERTIAL WAVES DAMPING ON ORBITAL DYNAMICS

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Abstract. Almost all regular satellites of Solar System giant planets exert tidal forcing at frequencies within the range for which inertial waves can be excited in their fluid envelope. Their damping, through their interaction with the turbulent friction of convection, provides an efficient mechanism of tidal dissipation that strongly depends on tidal frequency and leads to the orbital evolution of the satellites. We present in this work a first attempt to understand this evolution with taking hydrodynamics of planetary interiors into account.

Keywords: celestial mechanics, hydrodynamics, planet-star interactions, planet and satellites: dynamical evolution and stability

1 Introduction

Gravitational tides are a key element to understand the dynamical evolution of planets because of exchanges of angular momentum induced between orbits and spins. Since Goldreich & Soter (1966), this interaction has often been modelized by a constant quality factor Q calibrated empirically and bound to the friction inside the body. However, the dependence of Q with respect to the tidal frequency χ is nowadays progressively taken into account (e.g. for rocky planets see Efroimsky & Lainey 2007, refeered as Paper I). Indeed, most of the existing models assume a smooth variation of Q as a function of χ. But, Ogilvie & Lin (2004), refeered as Paper II, showed that dissipation due to inertial waves in convective fluid planets is written as a sum of resonant terms with a strong dependence on χ. Following Paper II, we then consider the impact of this resonant dissipation on the orbital dynamics of the tidal perturber. First, we give the general set-up. Then, dynamical equations are presented together with qualitative results. Finally, we propose a quantitative scaling law establishing the link between the internal physics of the central body and the orbital dynamics of the perturber.

2 Physical set-up

In Paper I, the authors studied the fall of Phobos on Mars using a coplanar, circularized, two body rocky system. Here, we modify their set-up by considering a fully convective fluid central body A with the mass of Mars. Let us quote M_A its mass, R_A its radius, k_2 its Love number, and $\mathbf{\Omega}_A$ its spin that we suppose to be constant in this work. The fluid is newtonian and viscous, with a density ρ and a kinematic viscosity ν. The satellite is assumed to be puctual of mass M_B, and orbiting with a mean motion n_B and a semi-major axis a.

To determine the energy dissipated by viscous friction acting on tidally excited inertial waves in A, we use the local model derived in Paper II where the rotating fluid is contained in a cartesian box of lenght L and submitted to the tidal excitation (see Fig. 1). Paper II showed that such a model already provides the main properties of tidal dissipation in rotating fluid planets.

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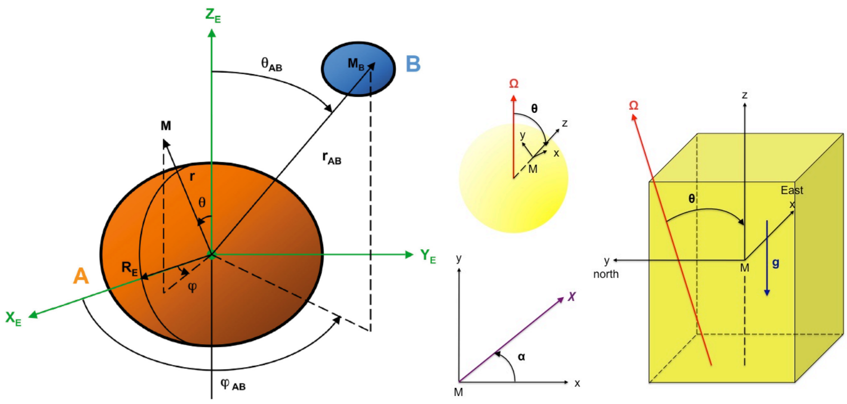

Fig. 1. Left: two-body system; here $\theta_{\rm AB} = \pi/2$ (figure taken from Mathis & Le Poncin-Lafitte 2009, courtesy Astronomy & Astrophysics). **Right:** The rotating cartesian box.

The dynamical equations are the same as in Paper I. First, we have for a:

$$\frac{da}{dt} = -\frac{3k_2 R_A^5 n_B M_B}{M_A a^4} Q^{-1}\left(\chi\right) \text{sgn}\left(\chi\right), \tag{2.1}$$

where χ is here the main tidal frequency $\chi = 2\left(n_B - \Omega_A\right)$ and Q is explicitly a function of χ. This dependence links the dynamical evolution to the internal structure, dynamics, and rheology of A.

Our local fluid model provides us the viscous dissipation in fluid, $D\left(\omega\right)$, which is proportional to $\omega\, Q^{-1}\left(\omega\right)$, $\omega = \chi/2\Omega_A$ being the normalized tidal frequency. Assuming that the box is located at the pole as in Paper II, the dissipation is expressed as a sum of resonant terms:

$$D\left(\omega\right) = D_0 \sum_{\{m,n\}\in\mathbb{N}^*\times\mathbb{N}^*} \frac{\left(m^2+n^2\right)\left|\tilde{\omega}^2\right| + n^2}{\left|\left(m^2+n^2\right)\tilde{\omega}^2 - n^2\right|^2} \left(m^2+n^2\right)\left|n f_{mn} - m h_{mn}\right|^2, \tag{2.2}$$

where the index m and n correspond to the wave-vectors in the azimutal and radial directions, respectively. We introduce in this formula a characteristic complex frequency, $\tilde{\omega} = \omega + iE(m^2+n^2)$, parametrized by the Ekman number of the fluid, $E = \nu/\left(2\Omega L^2\right)$. The external volumic excitation intervenes through the Fourier coefficients f_{mn} and h_{mn} describing the tidal forcing. The tidal dissipation strongly depends on tidal frequencies and presents resonances linked to the rheological parameters of the rotating fluid, *i.e.* the Ekman number.

Using the set of parameters of Paper I, we compute the evolution of the semi-major axis a over time with our fluid model and a constant Q. In Fig. 2 (left), we plot the spectrum of viscous dissipation as a function of χ. Next, in Fig. 2 (right), we see the difference between the evolution of a in the case of a Q constant model and the resonant one. The first one causes a regular fall while the fluid model induces an erratic evolution. Indeed, each time the system meets a resonant peak corresponding to a dissipation far higher than the background, the position of the satellite changes abruptly, and so does its mean motion simultaneously. This is the resonance locking identified in the stellar case by Witte & Savonije (1999).

3 Scaling law

As illustrated in Fig. 2 (right), the jumps in the evolution of the semi-major axis do not have the same amplitude. Indeed, it depends on the characteristics of the peak, which are themselves defined by the fluid properties because orbital dynamics and rheology are coupled through the shape of resonances. Equation (2.2) enables us to express the amplitude of a jump in terms of the main characteristics of the associated peak, namely its height H_p and width at mid-height l_p, which are functions of the Eckman number E. Near a resonance, one of the terms of the sum dominates all the others. Considering that the width of a peak is small compared to the distance which separates it from its nearest neighbours and that the numerator varies smoothly compared to the denominator,

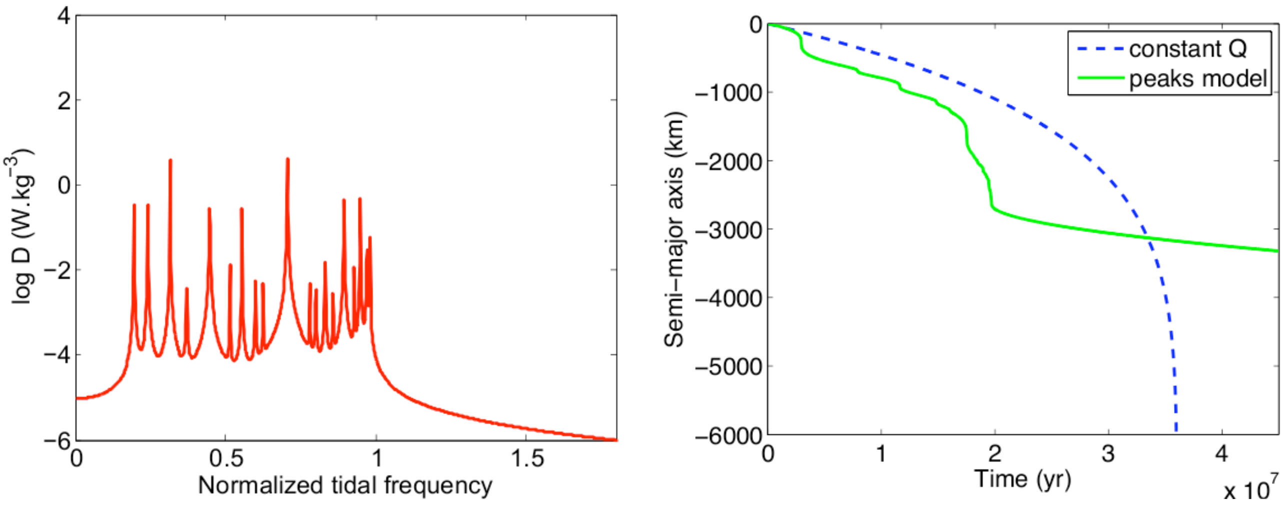

Fig. 2. Left: Resonant tidal dissipation spectrum as a function of the normalized tidal frequency assuming $m \leq 5$ and $n \leq 5$. The vertical axis is in logarithmic scale. **Right:** Evolution of the semi-major axis a of the satellite over time with a Q^{-1} proportionnal to the dissipation of tidal inertial waves (green curve) and with a constant Q factor (blue dashed curve).

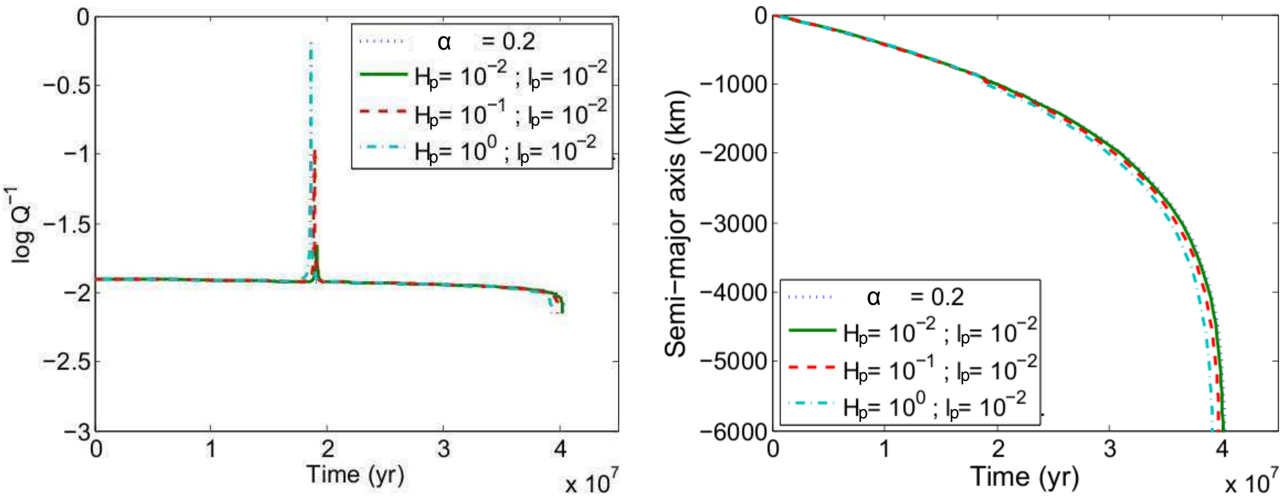

Fig. 3. Left: Evolution of the quality factor over time for various values of H_{p}. **Right:** Corresponding evolution of the semi-major axis. The grey dotted line corresponding to $\alpha = 0.2$ is superposed to the continuous green one except at the position of the peak for Q^{-1}.

we can isolate the synthetic resonant quality factor $Q_{\mathrm{p}}^{-1}(\omega)$:

$$Q_{\mathrm{p}}^{-1}(\omega) = \frac{H_{\mathrm{p}}}{\left[4\left(\sqrt{2}-1\right)\left(\frac{\omega-\omega_{\mathrm{p}}}{l_{\mathrm{p}}}\right)^2+1\right]^2}, \tag{3.1}$$

ω_p being the resonant frequency. Then, for a single peak, $Q^{-1}(\omega) = Q_0^{-1}(\omega) + Q_{\mathrm{p}}^{-1}(\omega)$, Q_0^{-1} being a smooth regular background varying slowlier than Q_{p}^{-1}. Assuming that the amplitudes of the variations caused by the peak on the semi-major axis Δa and the tidal frequency $\Delta\omega$ are such as $\Delta a \ll a$ and $\Delta\omega \ll \omega$, and considering that the resonance dominates , *i.e.* $Q_{\mathrm{p}}^{-1} \geq Q_0^{-1}$, we obtain the amplitude of Δa:

$$\frac{\Delta a}{a} \approx \frac{2 l_{\mathrm{p}}}{3\sqrt{\sqrt{2}-1}\,(1+\omega_{\mathrm{p}})}\left(\sqrt{\frac{H_{\mathrm{p}}}{Q_0^{-1}(\omega_{\mathrm{p}})}}-1\right)^{\frac{1}{2}}. \tag{3.2}$$

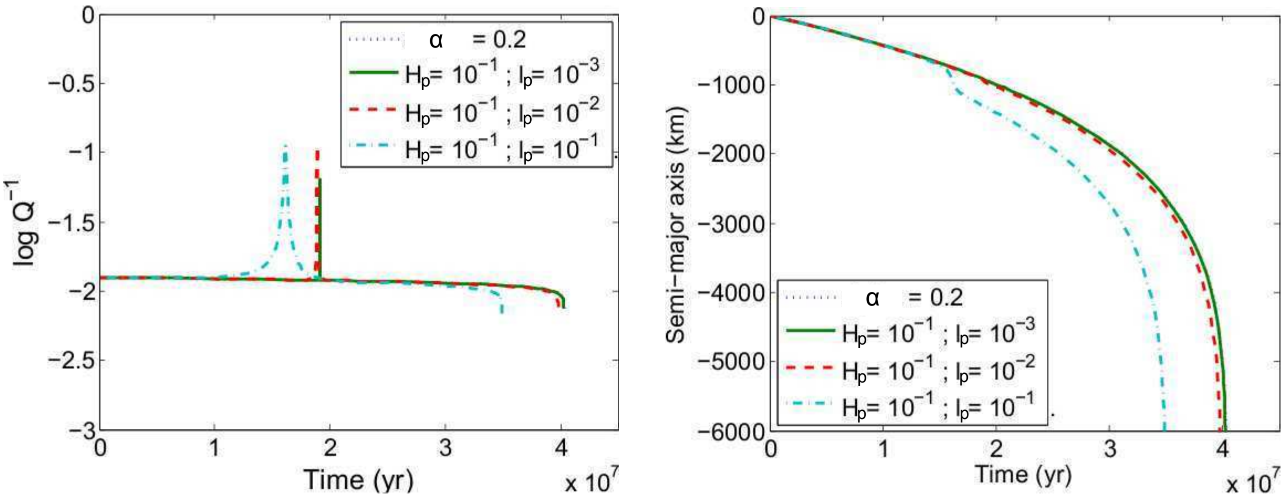

Fig. 4. Left: Evolution of the quality factor over time for various values of $l_{\rm p}$. **Right:** Corresponding evolution of the semi-major axis. The grey dotted line corresponding to $\alpha = 0.2$ is superposed to the continuous green one except at the position of the peak for Q^{-1}.

The impact of rheology is given by $\omega_{\rm p}(E)$, $H_{\rm p}(E)$, $l_{\rm p}(E)$ and $Q_0^{-1}(\omega_{\rm p})$. In a forthcoming article, we will explicitly give their expressions.

Following Paper I, where a synthetic smooth background $Q_0^{-1}(\omega) \propto |\omega|^{\alpha}$, α being a parameter, we simulate numerically the effect of a resonance on the semi-major axis of the satellite for different values of $H_{\rm p}$ and $l_{\rm p}$ as illustrated in Fig. 3 & 4. As predicted by Eq. 3.2, the simulations clearly demonstrate that the width of the peak $l_{\rm p}$ has a much stronger influence on the amplitude of the jump Δa than its height $H_{\rm p}$.

4 Conclusions

This work constitutes a first attempt to study tidal evolution of planet-moon systems with taking self-consistently hydrodynamical tidal dissipation in the central planet into account. It underlines the role played by inertial waves in convective rotating fluids and shows that dissipation may result from mechanisms narrowly bound to tidal frequency. A resonance between inertial waves and tidal excitation means a damping peak and a jump in the evolution of the satellite semi-major axis. Using the physical expression of viscous dissipation obtained in Paper II, we get a scaling law for jumps amplitude as a function of the height and the width at mid-height of the corresponding resonance. In a forthcoming article, we will detail their analytical expressions with respect to the fluid properties. Note that the obtained results can also be applied to star-planet systems and binary stars.

P. Auclair Desrotour was supported by the Paris Observatory (SYRTE). This work was supported by the Programme National de Planétologie (CNRS/INSU), the GRAM specific action (CNRS/INSU-INP, CNES), the Paris Observatory, the Campus Spatial de l'Université Paris Diderot, and the Emergence-UPMC grant (contract number: EME0911).

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