

CONTEXT

During the late stages of envelope accretion of the Giant Planets, the angular momentum of the infalling gas prevents it from being accreted directly onto the planet and it must proceed through an **accretion disk** [1]. It is within such a disk surrounding Jupiter, generally referred to as a subnebula, that the Galilean satellites Io, Europa, Ganymede and Callisto are thought to have formed [2]. However, neither the **structure** of the disk, nor the **origin** and **size** of the **solids** that eventually formed the satellites are currently constrained. To better understand the conditions of formation of the Galilean satellites, one must rely on the **observational constraints** so far available. The strongest of these constraints is the gradient in the **water** content of the moons [3]. Io is indeed completely dry, while ~ 8% of the mass of Europa is water and Ganymede and Callisto are made of half water half rocks.

In this context, we are developing a **numerical model** to track the evolution of solids of **different** sizes in the subnebula of Jupiter. The solids interact with the gas through **drag forces**, causing them to **drift** towards Jupiter or be entrained with the gas, and **turbulent diffusion**. During their evolution the solids are also heated by the surrounding gas and their ice sublimates. We thus are adding sublimation in order to track the **ice-to-rock ratio** of the solids as a function of time and position in the subnebula.

MODEL

The evolution of the density of solids is given by the **advection-diffusion** equation, which along any given direction in Cartesian coordinates reads [4]:

$$\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x} \left(\rho_p v_p - \rho_g D_p \frac{\partial \rho_p}{\partial x} \right) = 0$$

ρ_p, v_p, D_p : density, velocity and diffusivity of solid particles
 ρ_g : gas density

This equation is solved in **3D** using a **Monte-Carlo** particle tracking method. The position of particles subjected to the different **transport** processes is calculated over time. The new position of a given particle after a time step dt is then [5]:

$$x_i = x_{i-1} + v_{adv} \Delta t + R \left[\frac{2}{\sigma^2} D_p \Delta t \right]^{\frac{1}{2}}$$

R, σ^2 : a random number and the variance of the random distribution

$$v_{adv} = \frac{D_p \partial \rho_g}{\rho_p \partial x} + \frac{\partial D_p}{\partial x} + v_{p,x}$$

The velocity $v_{p,x}$ is obtained by integration of the equation of motion of the particle :

$$\frac{d\vec{v}_p}{dt} = -\frac{GM_{Pl}}{r^3} \vec{r} - \frac{1}{t_s} (\vec{v}_p - \vec{v}_g)$$

t_s : stopping time of the particle, i.e., characteristic time for gas to transfer angular momentum to the particle

NOTION OF TRACERS

When solving the advection-diffusion equation with a particle tracking algorithm, i.e. a Lagrangian approach, an important notion to understand is that of **tracers**. The particles whose motion is calculated at each time step are in fact tracers, meaning that they actually represent an **ensemble** of solids sharing roughly the same **size** and **position** within the disk.

In order to conserve the mass of the system each tracer is given a **fixed** mass budget M_t . This **mass budget** is defined by $M_t = M_T / N_t$, where M_T is the total mass in solids of the disk and N_t is the number of tracers used in the simulation. This way, a tracer has a physical meaning and can be easily converted into solids' mass. The **number** of actual solid particles represented by one tracer is not fixed and may **evolve** with time. It is defined as $n_p = M_t / m_p$, where m_p is the mass of one solid. This number is not fixed because when **sublimation** of ices or **collisions** are added in the simulation, the mass of the particles represented by the tracer evolves. Note also that n_p needs not be an integer but that a **significant** number of tracers should be used to properly solve the equation and to provide meaningful statistics. Here we generally used at least **10⁴** tracers.

RADIAL MOTION

Without Gas Drag

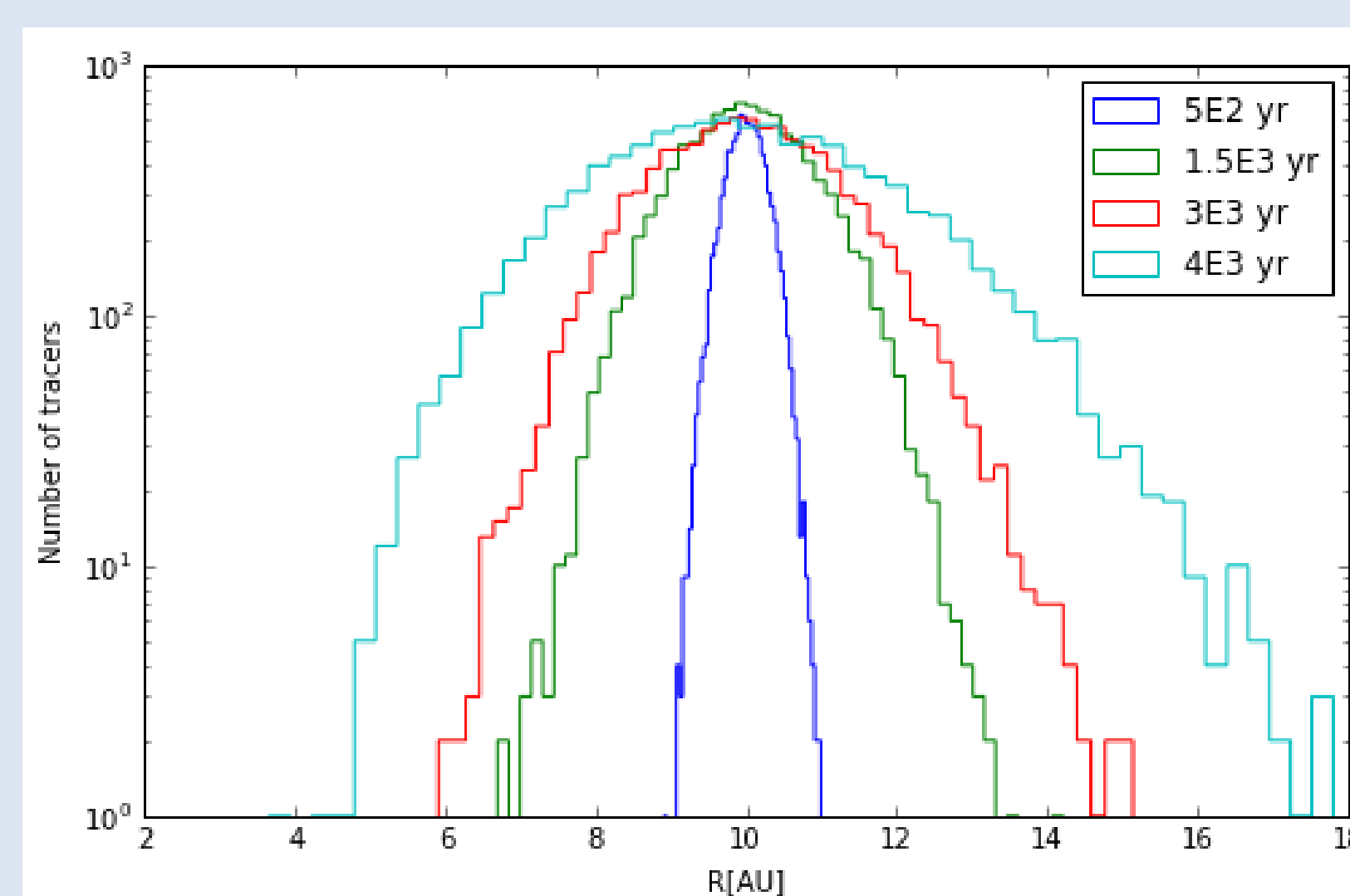


Figure 1. Diffusion of solid particles at the midplane of the disk without gas drag effects. 10^4 tracers were used, representing particles with a radius of 10^{-6} m. Note the asymmetry in the diffusion because of the gas density gradient.

With Gas Drag

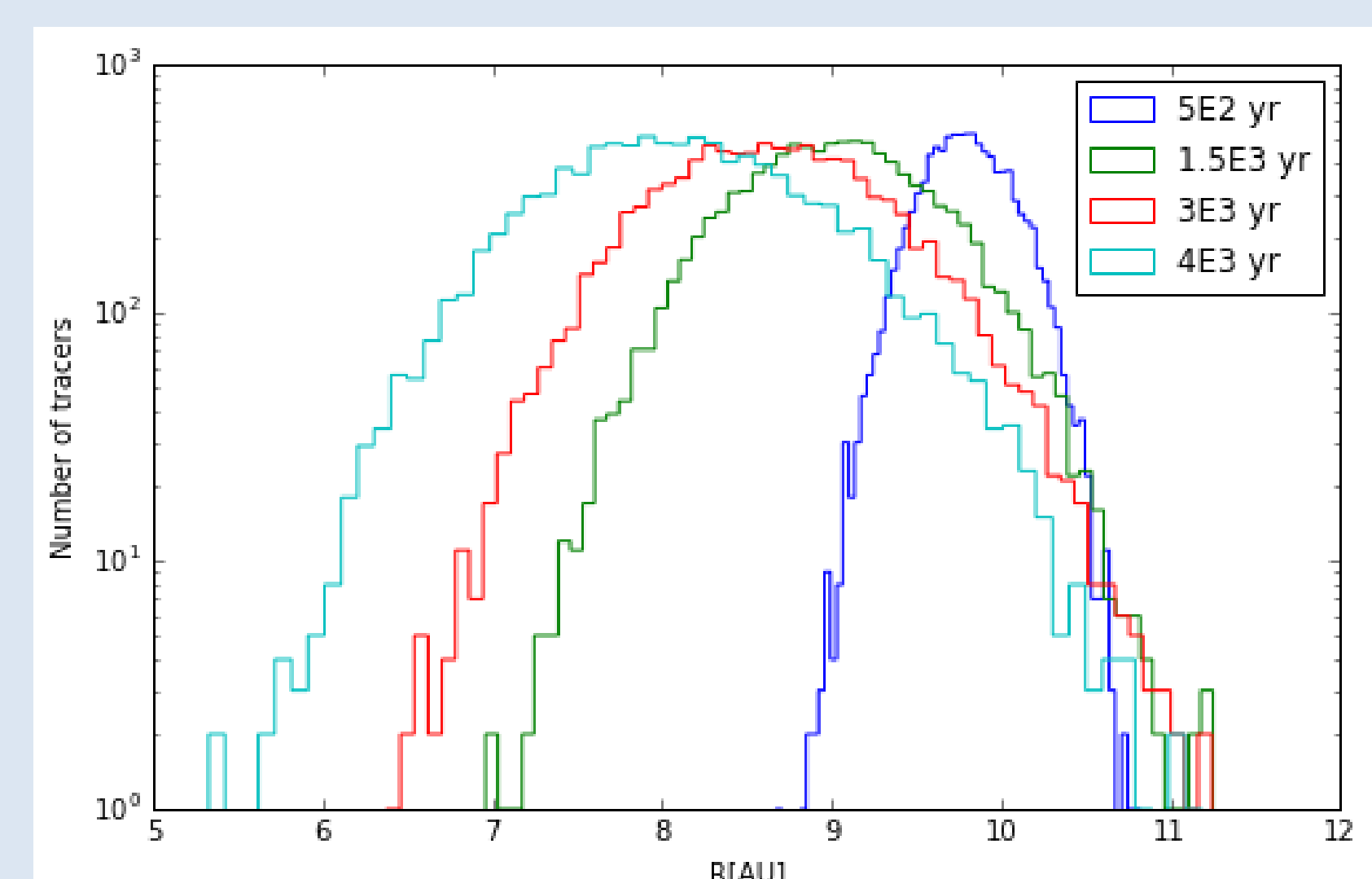


Figure 2. Same as figure 1 but with gas drag included in the simulation. The diffusion is still asymmetric but the collective behavior of particles is also clearly identifiable.

VERTICAL MOTION

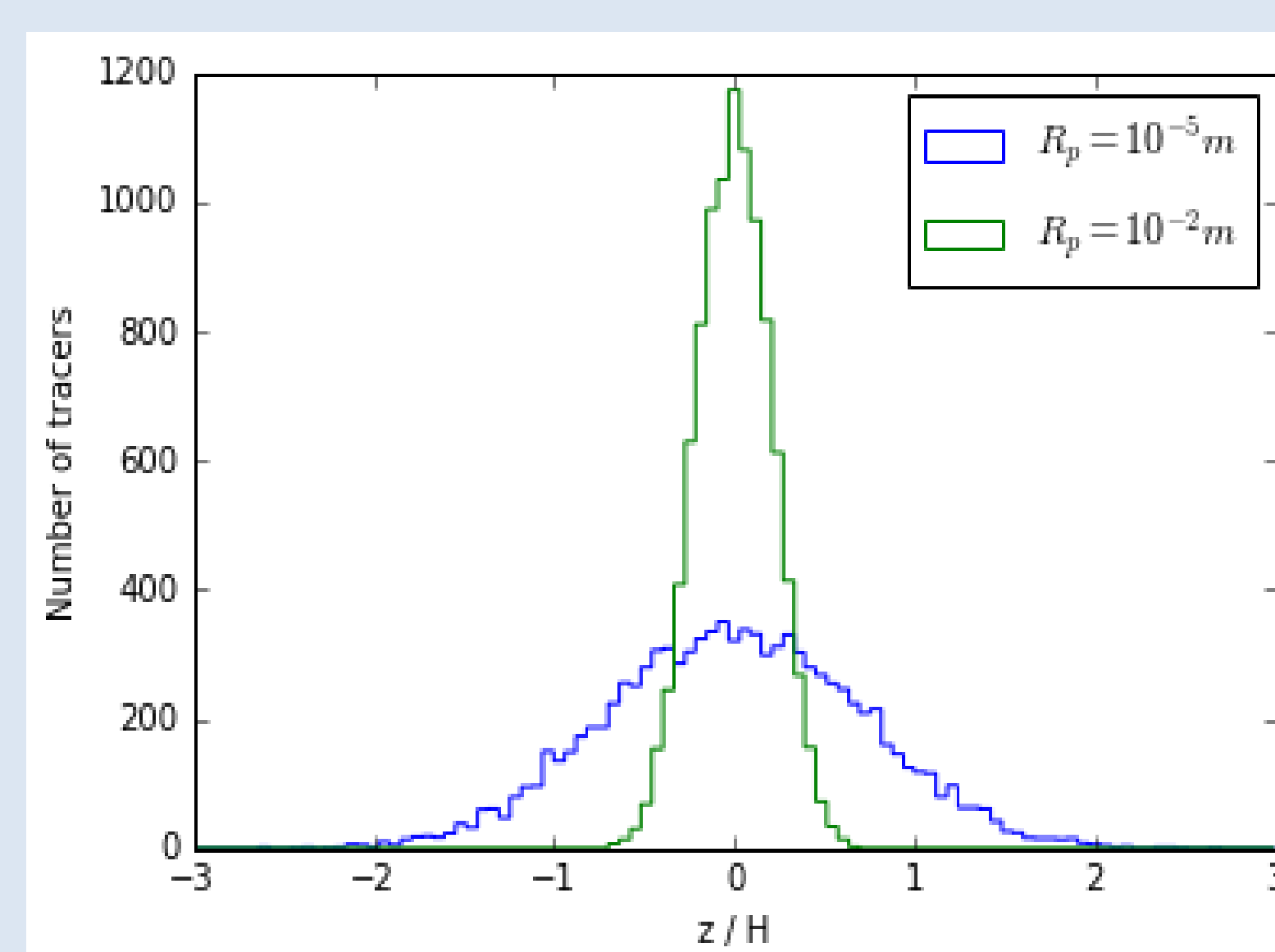


Figure 3. Settling of particles of different sizes. The simulations were run for 3×10^3 years with 10^4 particles. All particles were initially released at the midplane and evolved due to turbulent diffusion. The height z of the particles is normalized with the gas disk scale height H . The corresponding particles' scale height are $h_p \sim 0.7 H$ and $h_p \sim 0.2 H$ for particle sizes $R_p = 10 \mu m$ and $R_p = 1 cm$ respectively. Smaller particles are more tightly coupled with the gas and turbulent diffusion prevents them from being efficiently settled.

CONCLUSION & PERSPECTIVES

Here we present the **first** results of our simulation that includes several **transport** processes to describe the motion of **solid** particles within accretion disks. As we show, transport is influenced by **gas density** and **particle size**. Although not shown here, diffusion strongly depends on the **level** of **turbulence** as well.

The results presented are just the first step of this study. The aim is to track the **ice-to-rock** ratio of the particles during the **lifetime** of the subnebula (~1 My). To do so we are currently adding a module to determine the surface **temperature** of the particles and calculate the **sublimation** rate of water **ice**.

References

- [1] Tanigawa et al. 2012. *ApJ* 547, 47
- [2] Canup & Ward 2002. *AJ* 124, 3404
- [3] Sohl et al. 2002. *Icar*, 157, 104
- [4] Charnoz et al. 2011. *ApJ* 737, 33
- [5] Ciesla 2010. *ApJ* 723, 514