Turbulent environment: statistical and spectral methods for investigations of its characteristics

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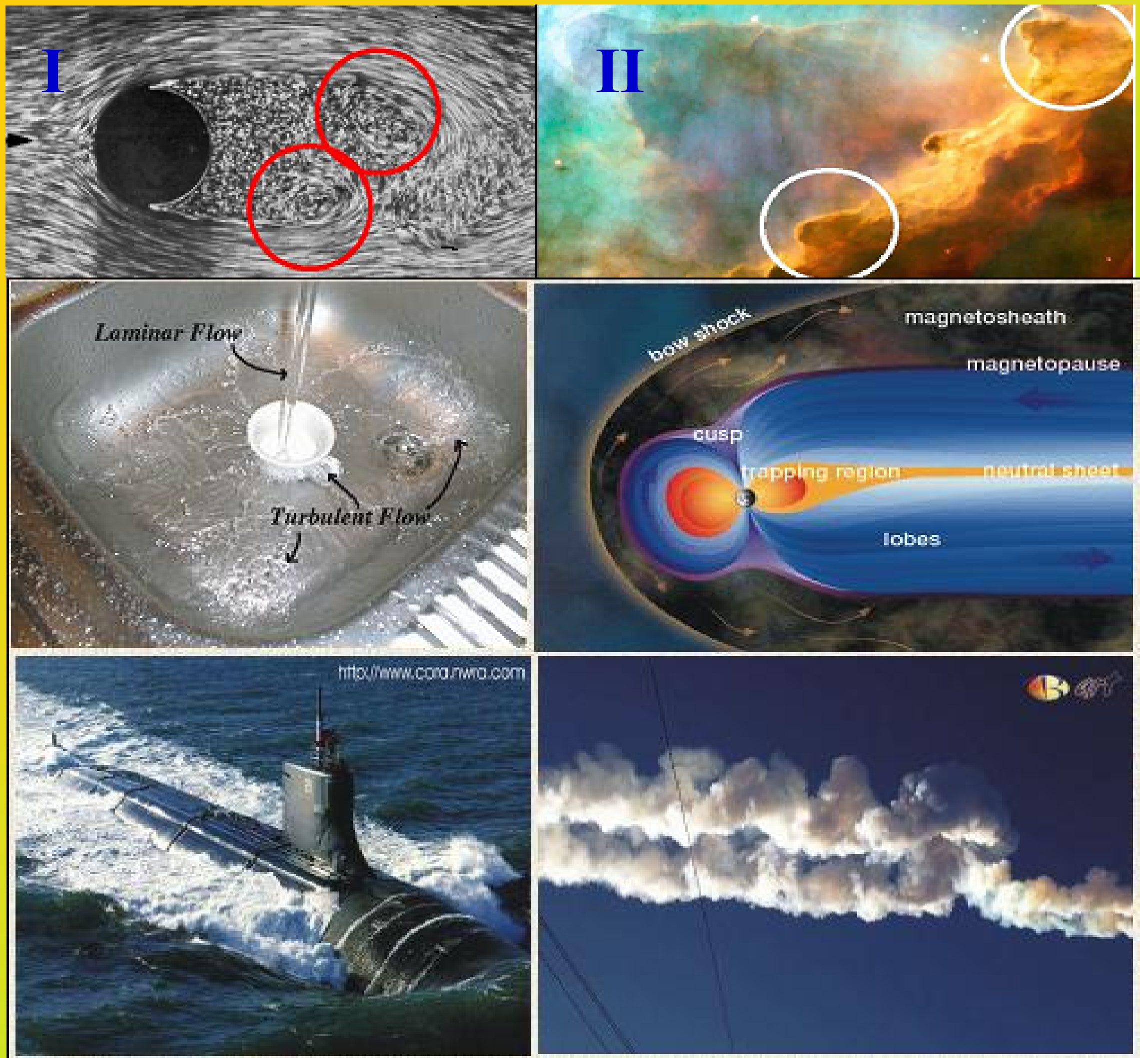
Examples of turbulent flows



I. Flow around a cylinder
II. Galactic clouds
III. Flow of water
IV. The region of magnetosphere
V. Flow around submarine
VI. Perturbations in atmosphere after the Chelyabinsk meteorite fall on 15 February 2013

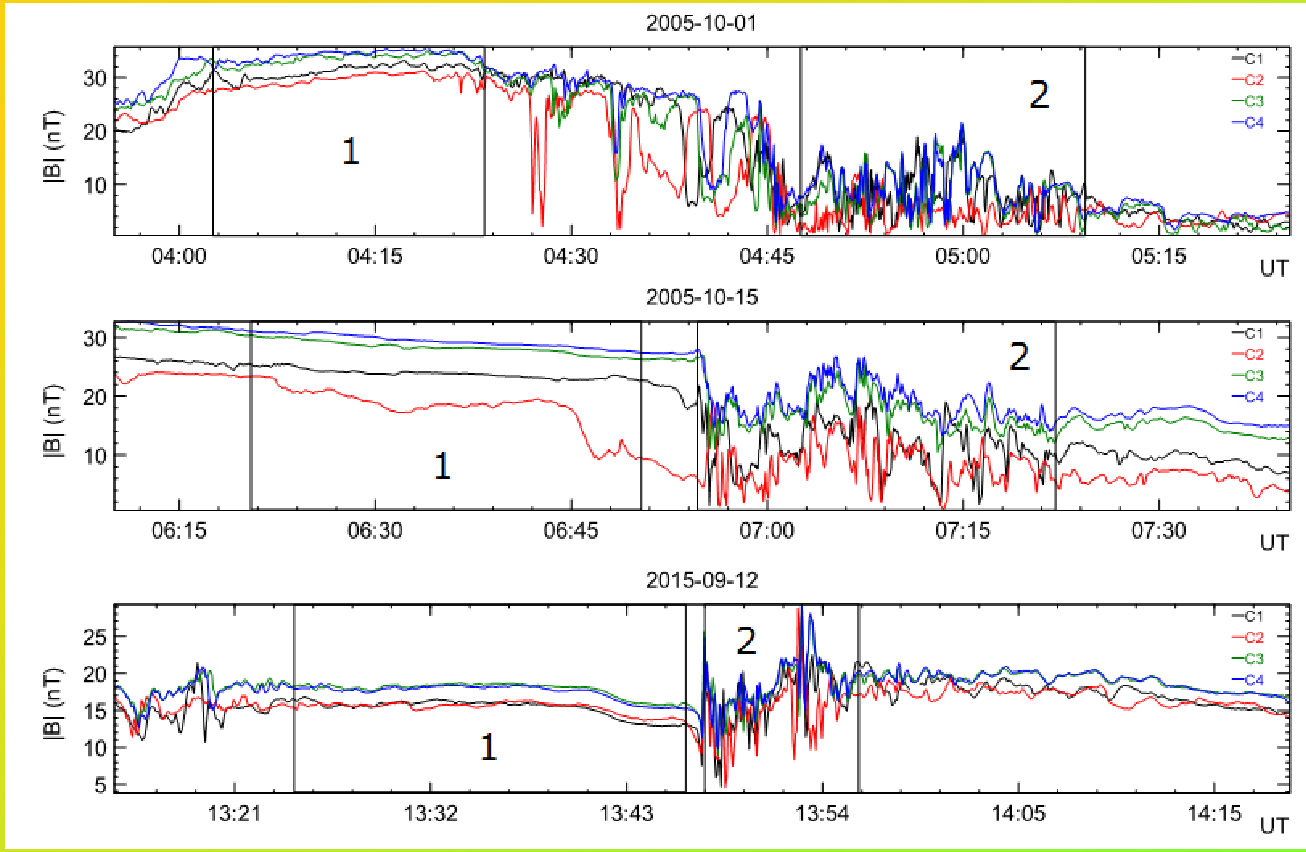
All these turbulent flows have common properties: 3D non stationary character and the presence in the flow both large (coherent) and small chaotic structures.

General information about the dipolarization

- The dipolarization (DP) phenomenon has been identified by the occurrence of large magnetic fluctuations predominantly around the neutral sheet of the magnetotail where $Bz >> Bx$, By.
- During DP, magnetic fluctuations reach the level in which dBz/Bzo is of the order one or larger, where Bzo is the Bz value before DP onset. It typically lasts for several minutes.
- During DP, the Bz component could become negative in spite of a strong background positive Bz component from the dipole magnetic field.
- It is accompanied by particle energization and intense fluctuating electric fields.
- The cross-tail current breaks up into filaments and may reverse its direction.
- The associated plasma flow pattern is not organized by the Bz polarity, unlike magnetic reconnection. In other words, tailward (earthward) plasma flow can occur in the presence of positive (negative) Bz. This is one of the distinctions between DP and magnetic reconnection.

Lui, A. T. Y.: Review on the Characteristics of the Current Sheet in the Earth's Magnetotail, in: Electric Currents in Geospace and Beyond pp. 155–175, John Wiley & Sons, Inc.

Examples of the analyzed values of the magnetic field



The peculiarities of the magnetic field fluctuations which were considered for moments prior to dipolarization (relative level of fluctuations ~ 0.05 - 0.4 for interval 1) and during the dipolarization of the magnetic field (relative fluctuation level ~ 0.8 - 1 for interval 2)

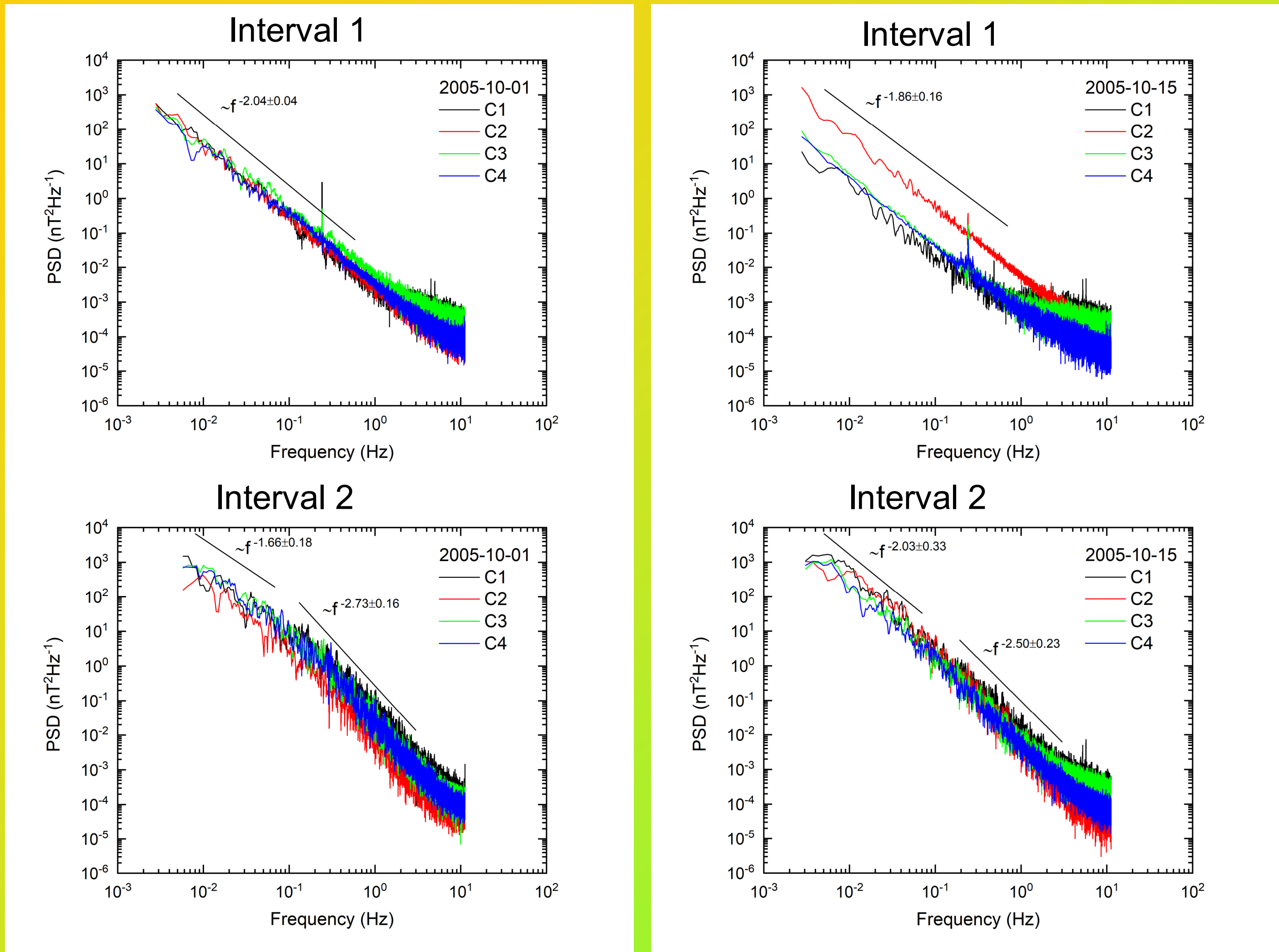
Spectral consideration: Power Spectral Density (PSD) analysis

The power spectral density (PSD) of the signal describes the power present in the signal as a function of frequency, per unit frequency.

- For Kolmogorov's theory of turbulence the dependence of the energy flux spectrum on the frequency can be written as
- $E_K(\mathrm{f}) \propto f^{-5/3}$.
- For model of Iroshnikov–Kraichnan the dependence of the energy flux spectrum on the frequency can be written as
- $E_{IK}(f) \propto f^{-3/2}$.
- For the Hall-MHD model: $E_{Hall\text{-}MHD}(f) \propto f^{-7/3} \div f^{-11/3}$,

- 7/3 when magnetic energy dominates kinetic energy, to - 11/3 when kinetic energy dominates magnetic energy.

Example of results of spectral analysis



The limitation of frequencies from above is due to the presence of instrumental noise, and from below, to the amount of data sampling and the edge effect of the smoothing procedure.

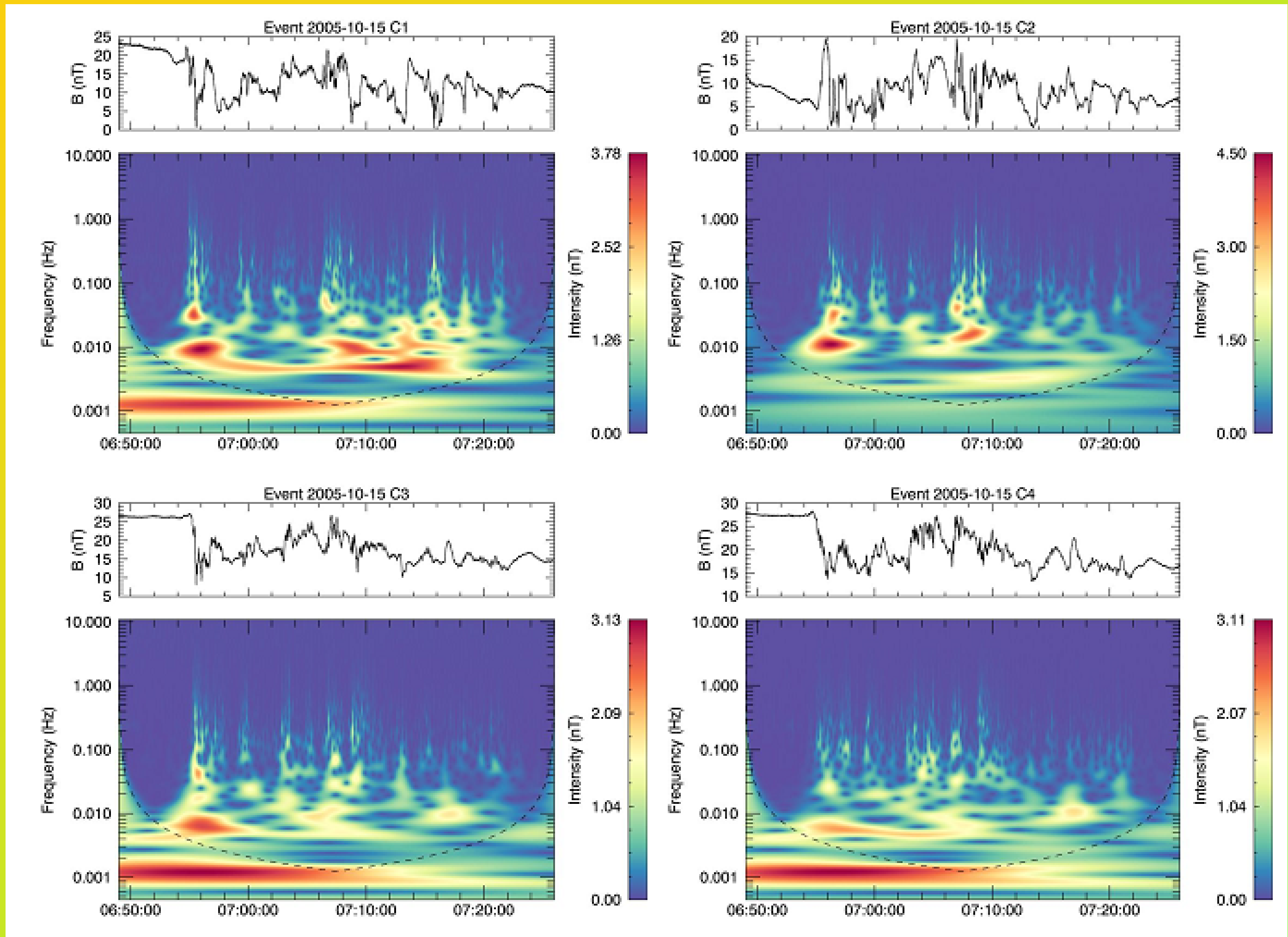
SF2A, Bordeaux, France

The results of spectral analysis

Spectral index

0.01 – 3 Hz				
Event	SC	f*	slope before	slope after
2005-10-01	C1	0.13	-1.6442	-2.8159
	C2	0.07	-1.8831	-2.4860
	C3	0.08	-1.5828	-2.8022
	C4	0.1	-1.5261	-2.8221
2005-10-15	C1	0.19	-2.1634	-2.5265
	C2	0.07	-1.5395	-2.7969
	C3	0.08	-2.1995	-2.3461
	C4	0.08	-2.1992	-2.3504
2014-11-09	C1	0.14	-1.6340	-2.8914
	C2	0.12	-1.6619	-2.4966
	C3	0.15	-1.5310	-2.8527
	C4	0.15	-1.5421	-2.8473

Example of the results of wavelet analysis

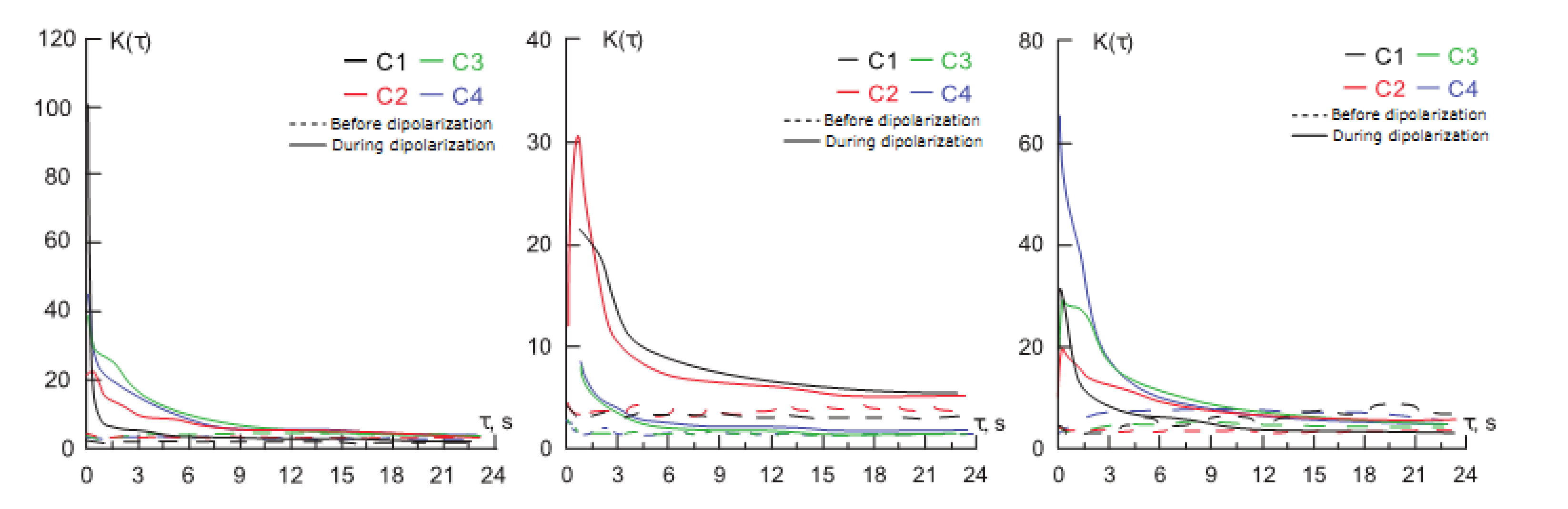


The cone of influence is shown by dotted line

Statistical method: kurtosis analysis

Kurtosis is expressed through the second and fourth moments by the relation: $K(\tau)=\frac{S_4(\tau)}{(S_2(\tau))^2}$, where, $S_4(\tau)=\left\langle\left|B(t+\tau)-B(t)\right|^4\right\rangle$, $S_2(\tau)=\left\langle\left|B(t+\tau)-B(t)\right|^2\right\rangle$.

For Gaussian distribution, $K = 3$.



(a) 2015-09-12 (b) 2005-10-15 (c) 2005-10-01

The results of kurtosis analysis of the magnetic field for different events

Statistical method: Extended Self-Similarity (ESS) analysis

Analysis of features of structure functions (moments of probability density function) of different orders of q in accordance with temporal interval.

Structure function is defined as [*Kolmogorov, 1941*]:

$$S_q(\tau) = \left\langle \left| B(t+\tau) - B(t) \right|^q \right\rangle \sim \tau^{\varsigma(q)}.$$

In the case of uniform isotropic Kolmogorov's 3D turbulence ***ζ(q) = q/3***.

For the model of Iroshnikov-Kraichnan [*Kraichnan, 1959; 1970; Iroshnikov, 1964*] **- *ζ(q) = q/4***.

Nonlinear functional dependence *ζ(q)* on the order *q* for experimental data is the result of intermittency processes.

In log-Poisson model, the structure function parameter exponent ***ζ(q)*** is defined by [*Debrulle, 1994*]:

$$\zeta(q) = (1-\Delta)\frac{q}{3} + \frac{\Delta}{1-\beta}\left[1-(\beta)^{\frac{q}{3}}\right],$$

where Δ and β are parameters that characterize the intermittency and singularity of a dissipative process.

For 3D isotropic log-Poisson model of turbulence $\Delta=\beta=2/3$ (SL) [*She and Leveque, 1994*]:

$$\zeta(q) = q/9 + 2\left[1-(2/3)^{q/3}\right],$$

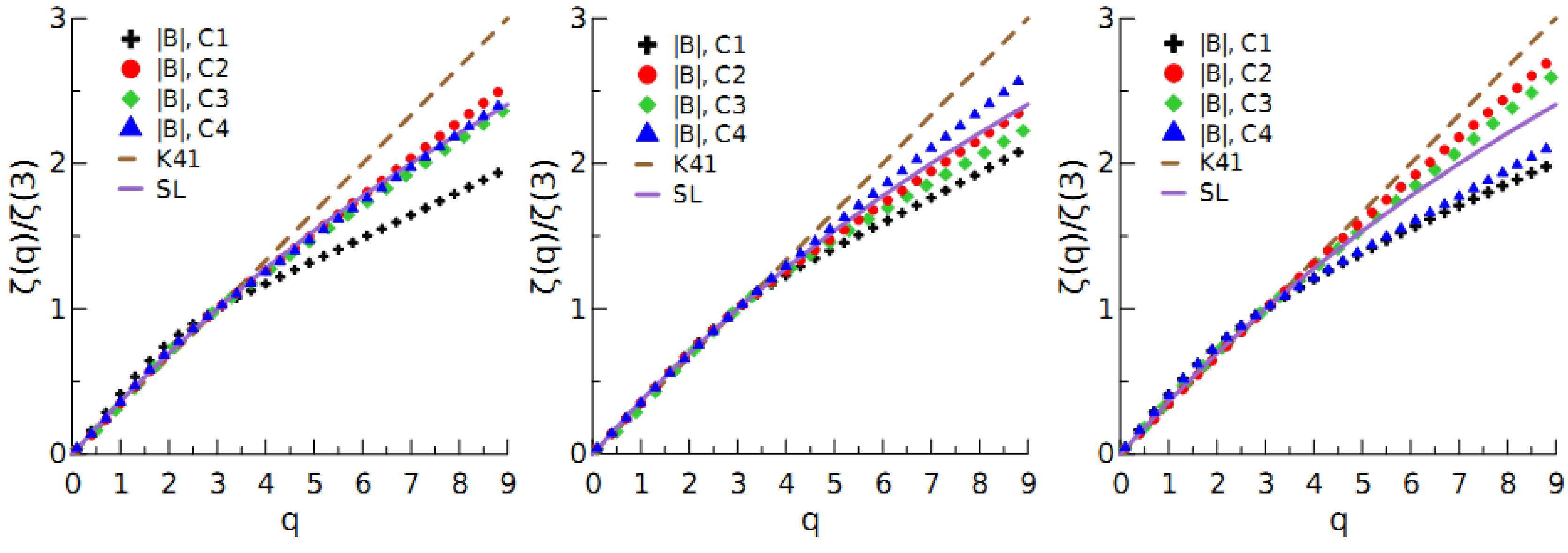
ESS analysis means the determination of the relative value of an exponent power for different orders of structure functions.

In the most common case for q-th and p-th orders the following relation exists [*Benzi et al, 1993*]:

$$S_q(\tau) \sim S_p(\tau)\ \tau^{\varsigma(q)/\varsigma(p)}.$$

Results ESS-analysis:

Comparison with 3D models of turbulent processes



(a) 2015-09-12 (b) 2005-10-15 (c) 2005-10-01

The results of ESS-analysis (during DP).
Ratio of the power of the q-th order structural function to the third order function power.
The experimental data for the magnetic field are marked with symbol; the solid line corresponds to the value calculated using the formula in the log-Poisson cascade model for $\Delta=\beta=2/3$ (SL), and the dotted line corresponds to the q/3 (K41)

Character of diffusion processes

Generalized diffusion coefficient [*Chechkin et al, 2002*] $D_f \sim \tau^{\Delta(1/\beta - 1)} = \tau^R$,
where Δ and β are powers of intermittence and singularity (Log-Poisson model)

For diffusion the dependence on time scale is absent – R=0.

For the anomalous diffusion

-in the case of R > 0, we have superdiffusion process,

-and for R < 0 we have subdiffusion process.

Parameters of the diffusion processes

Event	SC	ß	Δ	R
2015-09-12	C1	0.5± 0.029	0.77± 0.026	0.77
	C2	0.6± 0.018	0.45± 0.019	0.30
	C3	0.52± 0.091	0.43± 0.009	0.40
	C4	0.58± 0.016	0.46± 0.014	0.33
2005-10-01	C1	0.45± 0.015	0.41± 0.013	0.5
	C2	0.51± 0.013	0.2± 0.021	0.2
	C3	0.45± 0.024	0.21± 0.019	0.26
	C4	0.51± 0.026	0.54± 0.018	0.52
2005-10-15	C1	0.51± 0.015	0.67± 0.012	0.64
	C2	0.68± 0.025	0.72± 0.019	0.34
	C3	0.34± 0.027	0.22± 0.026	0.43
	C4	0.51± 0.026	0.24± 0.028	0.23

Conclusions

• The higher the PSD value the greater is the value of the height of the excess (kurtosis);
•Log-Poisson mode of turbulent processes with She-Leveque parameters corresponds to variations in the value of K(τ) in the range of 30-40
• The spectral indices correlate with the values of the diffusion coefficient.
•The critical scale for the processes is the ion-cyclotron frequency.
•Generalized diffusion coefficient increases with larger scales.
•The generalized diffusion coefficient is varied in the range of 0.2 - 0.77.
•The inverse and direct cascade processes can be formed in the magnetotail.
•The turn in the spectrum points to the presence of different scales of turbulent processes.
•Turbulent processes during of the dipolarization correspond to the Hall-MHD model turbulence.
•During dipolarization the large-scale and multi-fractal disturbances of the magnetic field are fixed, and the presence of inverse cascade processes also indicates the possibility of self-organization processes.

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