Current status of the modelling of the stellar structure and evolution of main sequence solar -like oscillating stars

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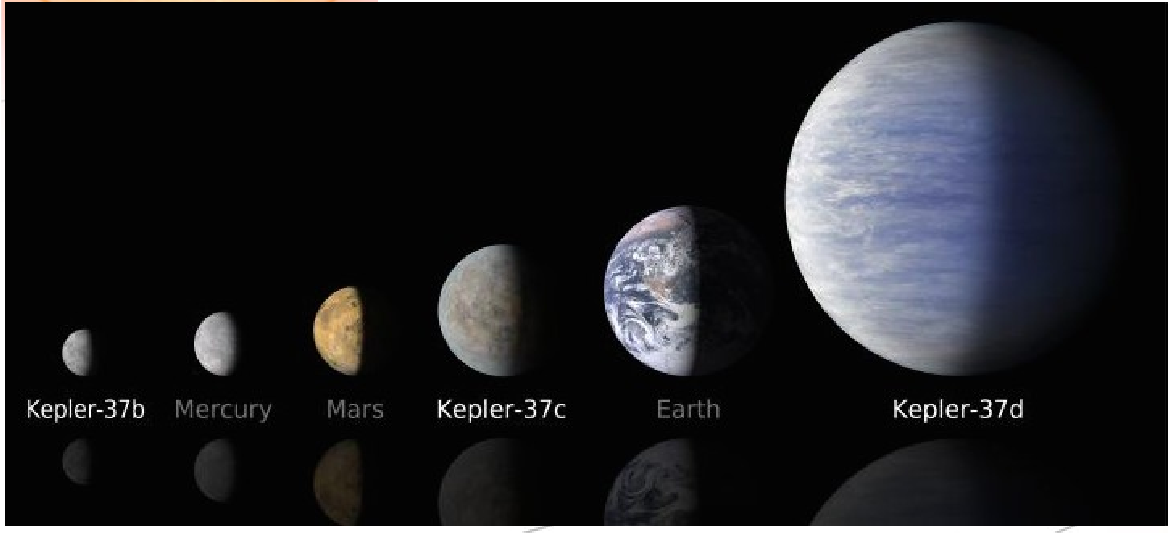
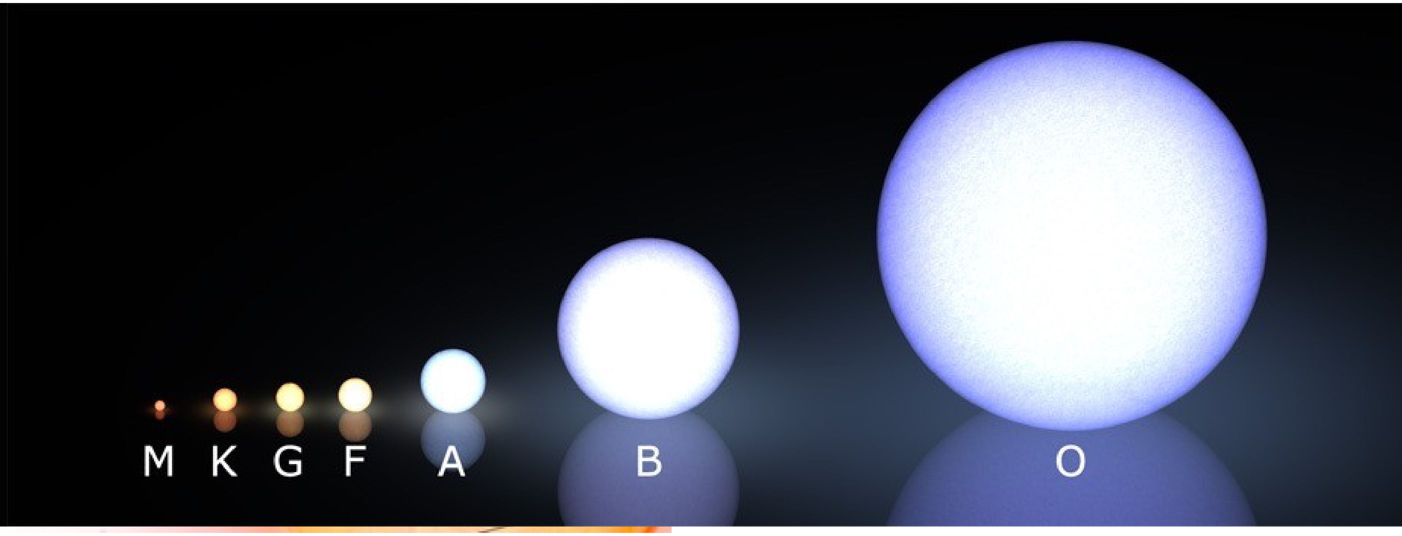
LESIA, OP

What do we need to know about the host star ?

Credit Magali Deleuil





Charactériser les exoplanètes …

excellente connaissance de l'étoile requise !

Masse + rayon → densité moyenne
gaseuse vs rocheuse, structure

→ masse et rayon de l'étoile

Composition → formation

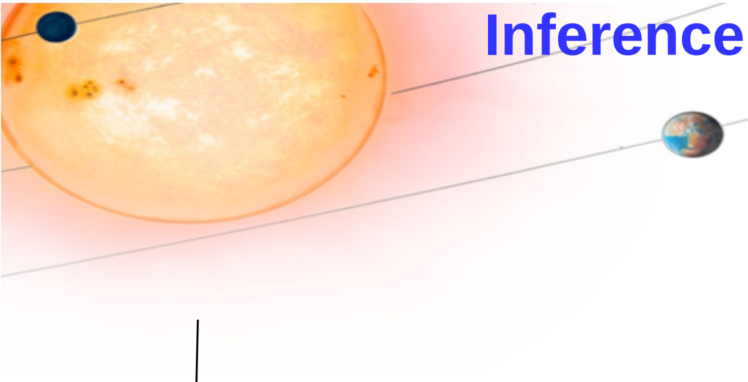
→ composition de l'étoile

Propriété de l'atmosphère
habitabilité

→ propriétés de l'étoile, insolation

Age → évolution
évolution des systèmes planétaires

→ age de l'étoile



Inference techniques for stellar mass, radius and ages

Input : - data
- parameters

Inference

Stellar models

Stellar models

HR
isochrone

One -to-one relation
Li-age
Lx-age
Prot-age

Seismology
Forward and inverse

Stellar properties
M,R ,A

Sources of uncertainties in deriving stellar properties

- Propagation of observational errors → statistical error / precision
- Resolution power of the observables → systematics
- Biases due to the inference technique → systematics
- Biases due to the morphology of the grid for grid-based technique → systematics
- **Uncertainties on the physics of stellar models** → systematics
  - → known , can be partially improved or compensated by varying free parameters
  - → known , modelling in progress if possible
  - → unknown : needs benchmarks for diagnostics and hint on the missing process

→ Observational errors have decreased at the level where systematics dominate as sources of uncertainties

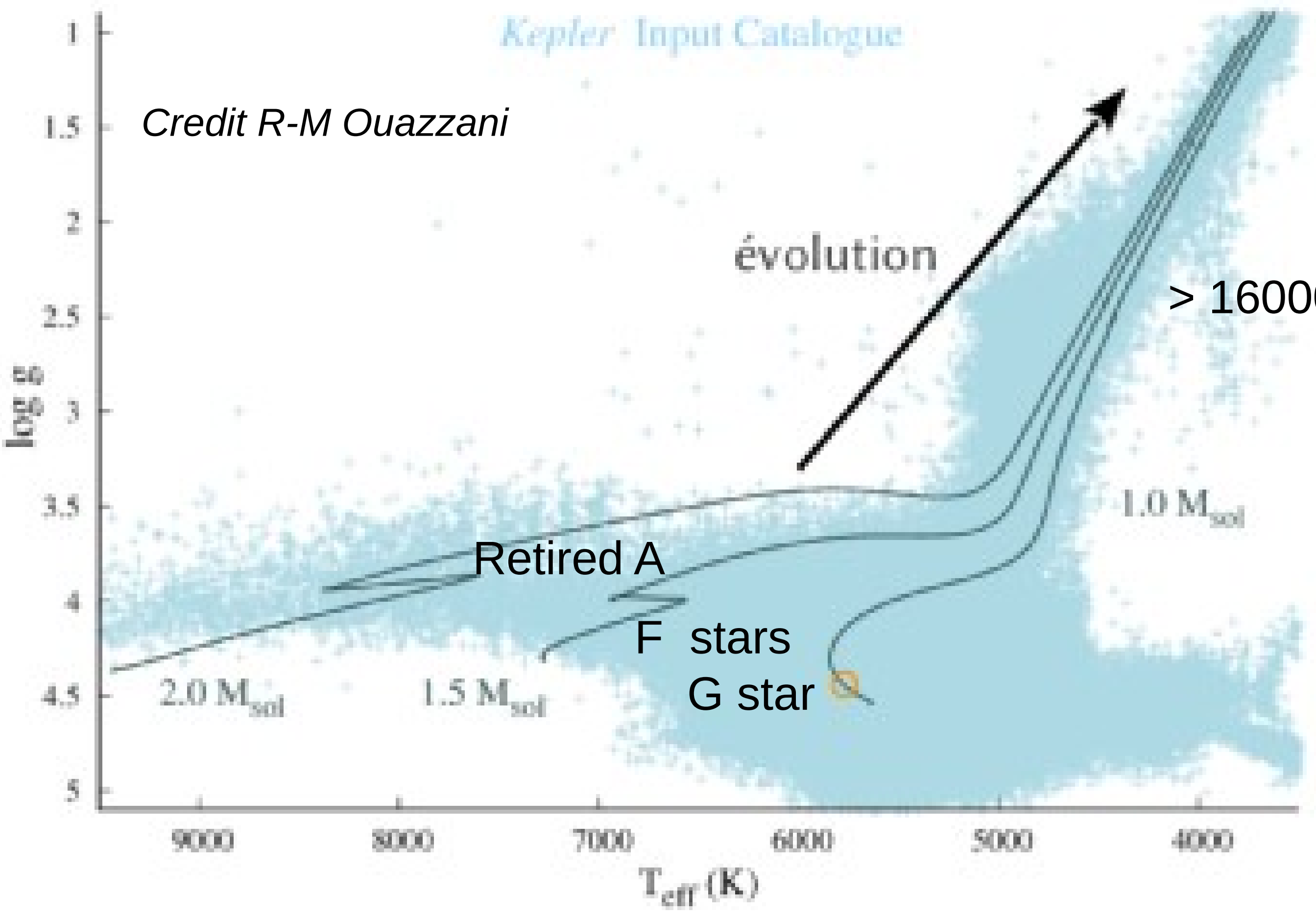
→ Caution : distinction between precision and accuracy

From now on, focuse on sources of uncertainties in stellar modelling for M,R,A inferences for stars with solar-like oscillations

Stars with solar -like oscillations

Wealth of high quality data mainly due to
CoRoT (Baglin et al., 2006)
Kepler (Borucki et al., 2010)
K2 (Howell et al., 2014)

Here focuse on **main sequence** stars with solar like oscillation :
Mass range [0.7<~ - ~ <1.5]M_{sun}



Stellar mass, radius age inferences

OUTPUT : Ajusted parameters : Mass, radius, age

INPUT : Observational constraints : assume Teff, [Fe/H] (assimile to [M/H]), seismic data . In specific cases, L--> R , M

INPUT : Free parameters : initial chemical composition **(**Y_{ini}, Z_{ini}, mixture) ; α_{MLT} , α_{ov} , ...

Learning sets :

- Simulated samples of stars
- CoRot : ~ 10 * (*Michel+2008, Noels & Deheuvels 2016 , CoRoT Legacy book)*
- Kepler : ~ 500 * with detection
  66 * → Kepler legacy
  +32 → KOI hosting planets
  + 16 000 RG

Chaplin+ 2014, Lund+2016, Davies+2015

Stellar mass, radius age inferences from seismic data : detailed modelling of individual stars

Many studies of individual stars

Here illustrations taken from studied of 4 specific stars

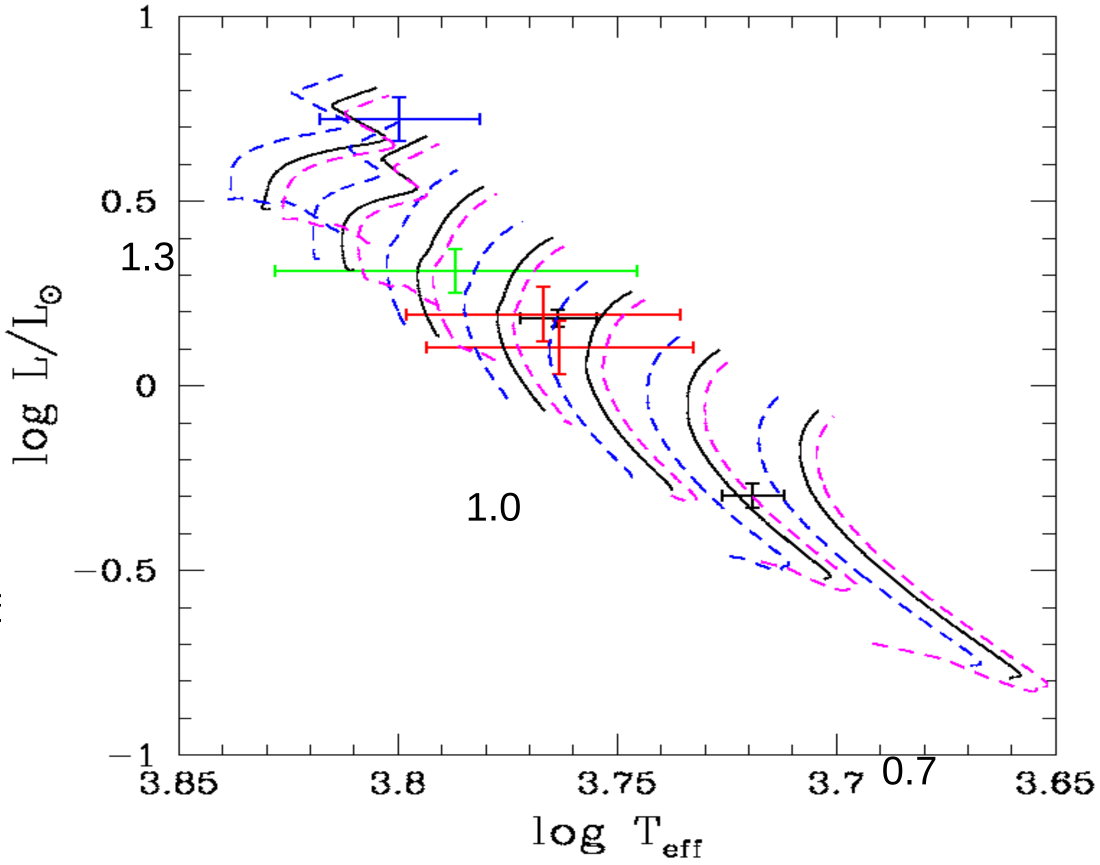
Kep 21 $1.408^{0.021}_{-0.030}$ M_{sun} *(Silva Aguirre 2015)*
planet host
Evolved : end of MS -subgiant depending on core overshoot

[Fe/H] =-0.03± 0.010

HD52265 : 1.14−1.32 M_{sun} *(seismic, Lebreton & Goupil 2014)*
planet host
[Fe/H] =0.22 ± 0.05
Existence of a small convective core ?

Alpha CenA(B) 1.1055 ± 0.0039 M_{sun} *(binary, Kervella+ 2016*
[Fe/H] =+0.24 ± 0.03
Existence of a tiny conv.core ?
Alpha Cen B is a planet host

16 CygA(B) 1.05-1.13 M_{sun}(seismic)
16 CygB is a planet host
[Fe/H] =0.096 ± 0.026



Major sources of uncertainties in stellar modelling for (M,R,A) inferences :

1) First dominant source of uncertainties on mass, radius and age is the **scatter of values for the free parameters** *(Lebreton & Goupil 2014)*

- initial helium content Y_{ini}

Impact of uncertainties on free parameters values: initial helium content

Mass - initial helium degeneracy
(*Metcalfe 2009, Baudin2012,Lebreton,Goupil2014)*

M-Yini degeneracy can hamper the precise, accurate mass determination

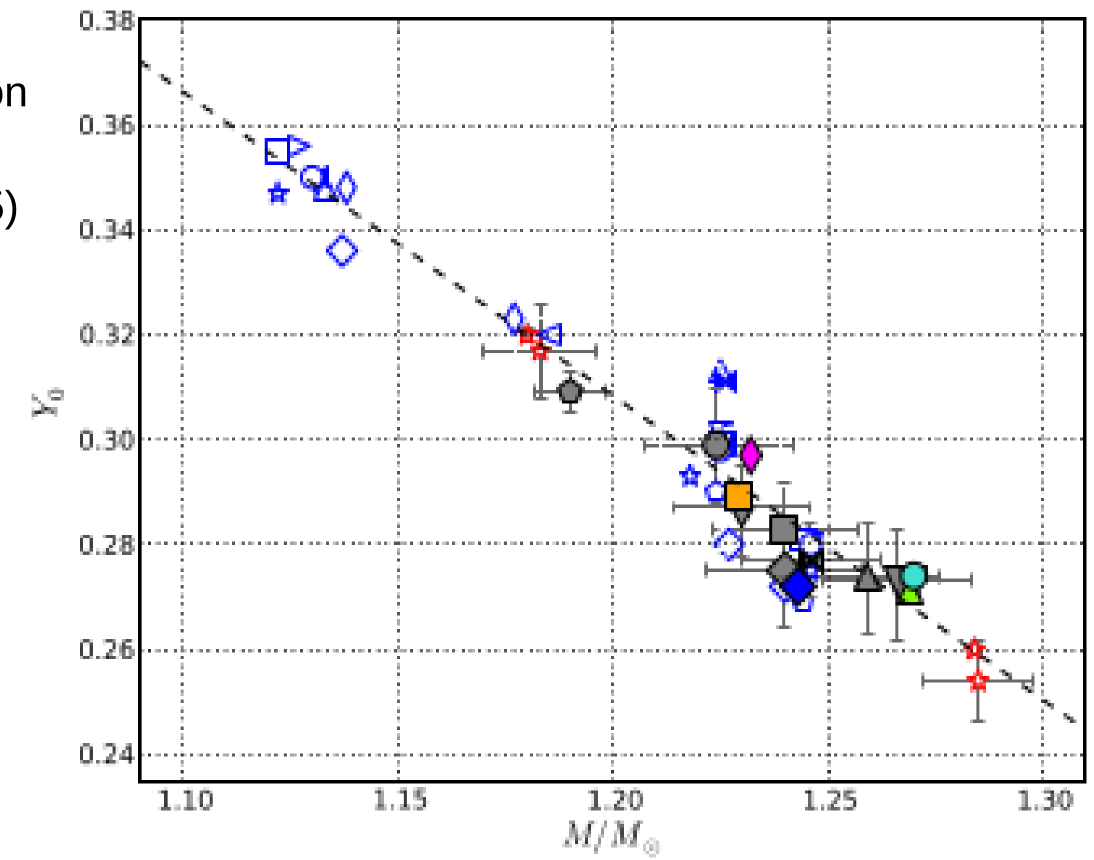
Ex: HD 52265 a G0V solar-like oscillator MS bright star (V= 6.3 ±0.005)

Observational constraints :
Teff , metal rich [Fe/H]=0.22 ± 0.05
Hipparccos distance → L → R/R_{sun} = 1.28 ± 0.06
Seismic data (4 months CoRoT , Ballot+2011)

Ajusted parameters : Mass, age

Free parameters : Y_{ini}, α_{MLT}, $(Z/X)_{ini}$, α_{ov}

HD52265



Inference: Y_{ini} = 0.24 – 0.28 and ΔY/ΔZ in the range 0.4−2.3.

→ M/M_{sun} 1.18−1.28 i.e. ΔM/M ~10 %

Lebreton &Goupil (2014)

Impact of uncertainties on free parameters values : initial helium content

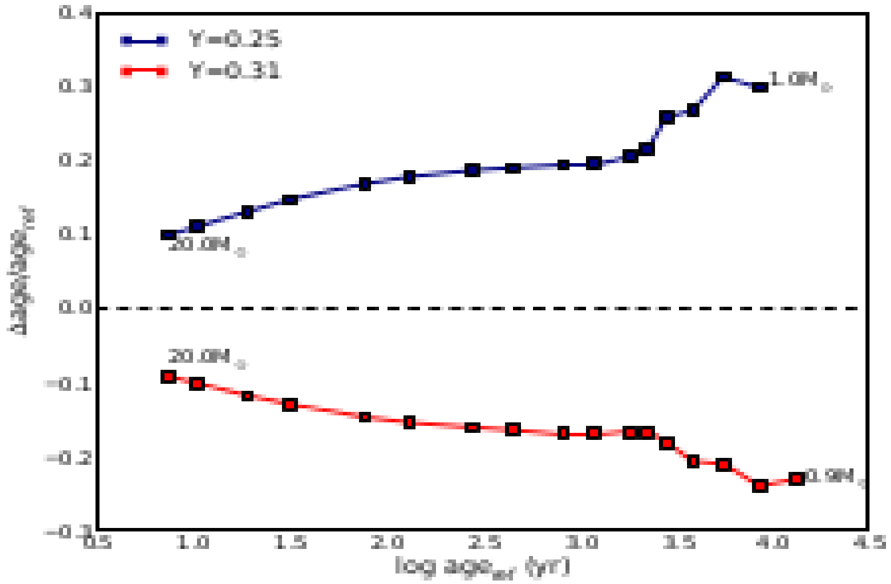
Correlation mass-age relative errors obtained from grid-based inferences on simulated samples of stars

Uncertainties on Y_{ini} → uncertainties on the mass M → uncertainty on the age , everything else fixed

Impact on age at turn-off

Difference with reference value Y_{ini} = 0.27

Yp = 0.248



Lebreton, Goupil, Montalban 2014

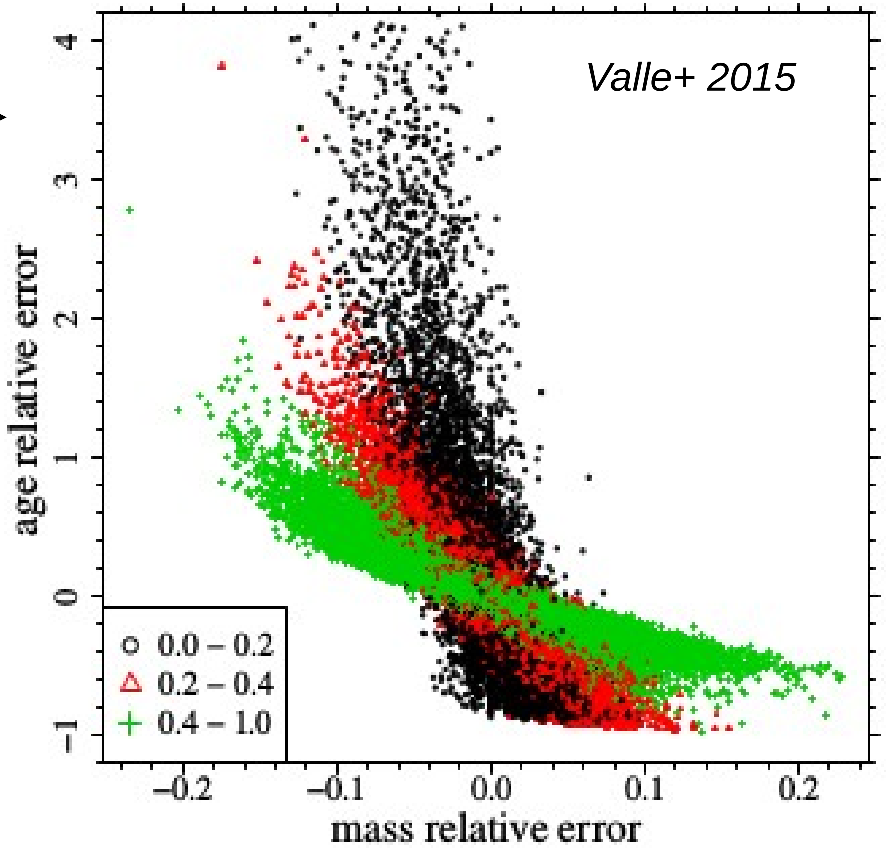


Fig. 4. Correlation between age and mass relative errors. The black circles correspond to models with relative age in the range [0.0; 0.2], the red triangles to relative ages in the range [0.2; 0.4], and the green crosses to relative ages in the range [0.4; 1.0].

Impact of uncertainties on free parameters values : initial helium content

Determining Yini → several options :

- Use the calibrated solar value $Y_{ini, sun}$ as a fixed value

- Initial helium content Yini from galactic enrichment law ΔY /ΔZ $Y_{ini} = Y_p + (\Delta Y / \Delta Z)\ Z$

  - Large scatter in ΔY / ΔZ *(Gennaro+2010)*

  - Usually assumed *1< ΔY /ΔZ <3*

  - *Ex : Silva Aguirre+17 BASTA grid-based inference of MRA of the Kepler Legacy sample : ΔY/ΔZ=1.4 (GS98) set fixed*

- Yini can be inferred as a free parameter with M,R,A → ΔY /ΔZ can also be deduced

  ΔY /ΔZ depends on adopted solar mixture and the star

  Ex : CenAB (*Joyce & Chaboyer 2018, AGSS09*) : ΔY /ΔZ = 0.90± 0.12

  When separating by star, Cen A gives ΔY /ΔZ = 1.08, on average, and Cen B gives ΔY /ΔZ = 0.72.

- Seismic glitch inference (*Houdek & Gough (2007), Verma 2016)*

  - surface helium of the actual star , not Y_{ini}

  - still model dependent (Y_{ini} not directly measured)

- Inversion for 16 Cyg A (*Buldgen+2016)*

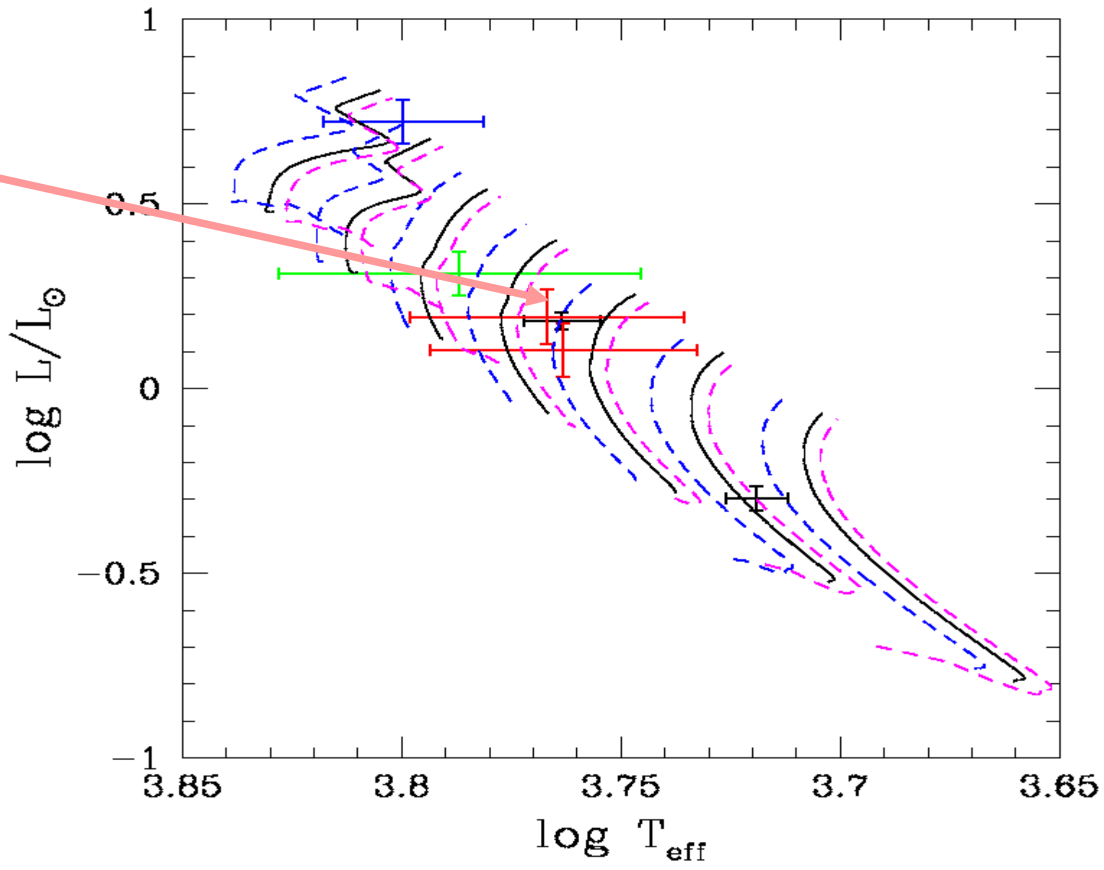
Impact of uncertainties on free parameters values : initial helium content

16 CygA a G0V solar-like oscillator MS bright star

Observational constraints :
Teff , metal rich [Fe/H]=0.22 ± 0.05
Interferometry R = White et al. (2013)
Seismic data (Kepler , Davies+ 2015)
Surface helium seismic constraint (Verma+2014)

Ajusted parameters : Mass, age

Free parameters : α_{MLT}, $(Z/X)_{ini}$



Inference leads to a lower scatter for M,A (*Buldgen+2016)*
M/M_{sun} = 0.97 - 1.0 ,
R/R_{sun} = 1.188 - 1.200
A = 7.0 - 7.4 Gyr

When the surface helium constraint (from Verma+ 2016) is removed : 1.09 M_{sun} and an age of 7.19 Gyr compatible with similar to the results from Metcalfe et al. (2012).

To remove this degeneracy, highly precise oscillation frequencies are required → brightest stars

Major sources of uncertainties in stellar modelling for (M,R,A) inferences :

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- initial helium content Y_{ini}
- solar chemical relative abundances (« mixture »)

Solar chemical « mixture »

Chemical 'mixture' : relative chemical abundances of heavy elements Stars chemical composition most often defined with respect to the solar composition : same mixture as the Sun

– Revision of the solar chemical mixture (1D --> 3D)

GN93 → GS98 → AGSS09 → AGS15

leads to

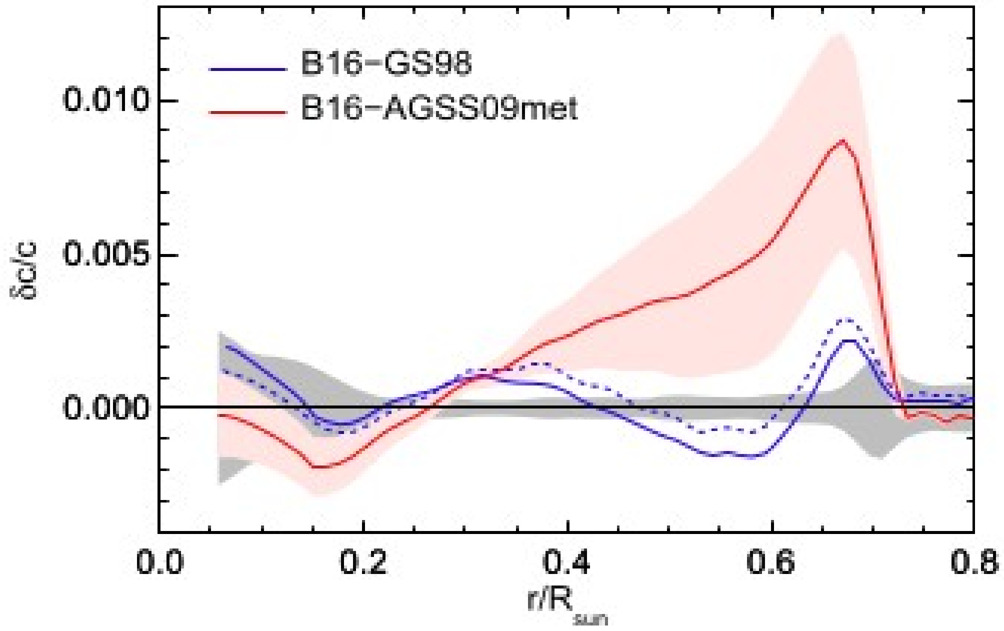
a high Z solar mixture → a low Z solar mixture

– GN93 : a good agreement between the seismic Sun and standard solar model with GN93 except for a very localized region below the ZC

– AGSS09 : more physically justified but a more general mismatch

	GN93	GN98	AGS05	Caff08	AGSS09	Lod09
$(Z/X)_\odot$	0.0245	0.0229	0.0165	0.0209	0.0181	0.0191

Vinyoles et al 2017 The Sun

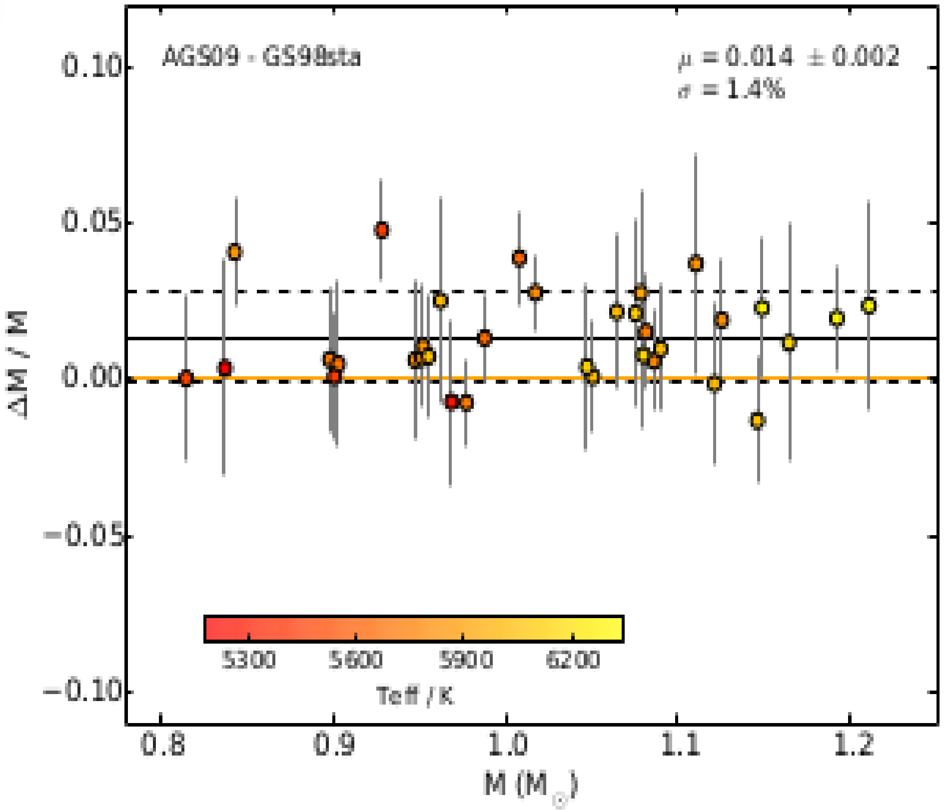


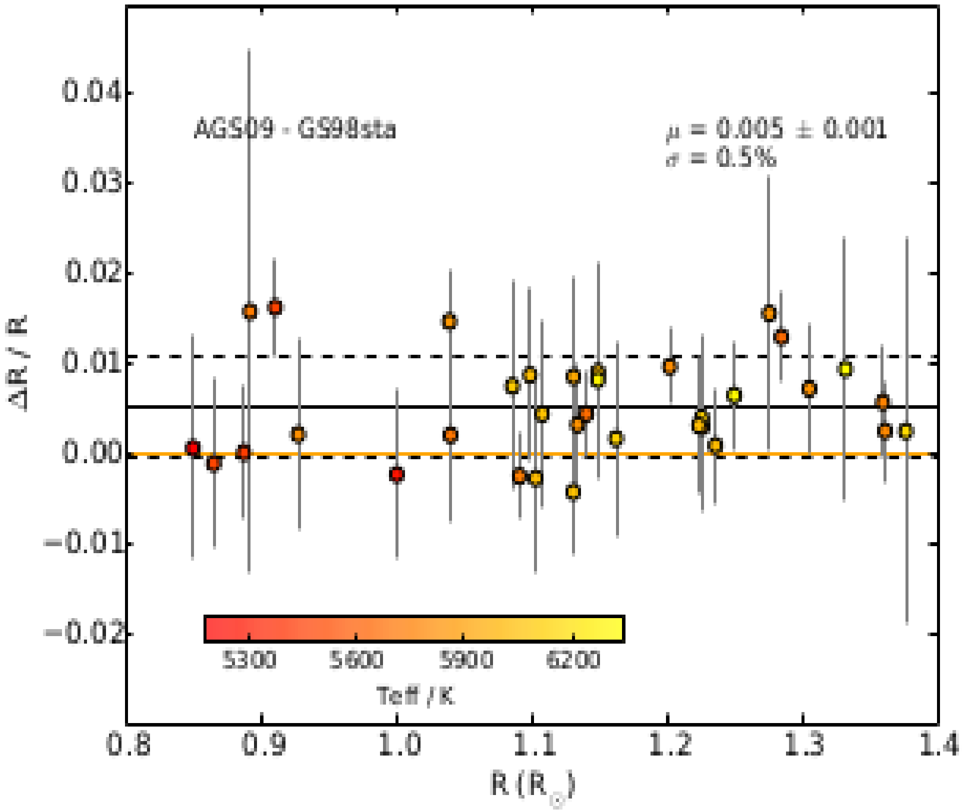
Difference in inverted sound speed between observations modelsB16
B16-GS98 computed with GS98
B16-AGSS09 computed with AGSS09
Increasing opacities by 7 % below the ZC does not solve the pb

Solar Mixture : GS98 → AGSS09

Nsamba+2018

32 stars from the Kepler legacy sample





Major sources of uncertainties in stellar modelling for (M,R,A) inferences :

1) First dominant source of uncertainties on mass, radius and age is the **scatter of values for the free parameters** *(Lebreton & Goupil 2014)*

- initial helium content Y_{ini}
- solar chemical relative abundances (« mixture »)
- mixing length

Impact of uncertainties on free parameters values : mixing length

1D modelling of superadiabatic turbulent convection → Mixing length is a free parameter

Determining the mixing length : several options :

- **Inferences using only classical parameters** (L, T eff and surface [Fe/H]) as constraints → the mixing length must be fixed.

  - Most often calibrated solar value → errors bars (error propagation + internal precision only) are artificially small

  - if α_{MLT} fixed taken within a (reasonable) range→ signifcantly large scatter in ages results.

Ex : $\Delta\alpha_{MLT}$ = 20 % around $\Delta\alpha_{sun}$ → the age changes by more than 50 %

- **Inferences using classical parameters and seismic data** as constraints → α_{MLT} is inferred in addition to M,R,A

Ex : $\Delta\alpha_{MLT}$ < ~ 4 % around α_{sun} → the age changes decrease to 10 %

- Taking the value from **empirical relations** *(Bonacca 2012, Viani+ 2018, Creevey + 2017)*

- Taking the value from **3D hydrodynamic convection** (*Trampedach+ (2014), Magic+ (2015)*)

Impact of uncertainties on free parameters values : mixing length

α CenAB : highly precise characterisation *(Kervella+ 2017, Porto de Mello+ (2008))*
+ seismic data → Yin, Zin, alpha_MLT, age to be ajusted

Joyce & Chaboyer 2018 : five different sets of assumptions about the physics
(150 000 tracks : > 15 millions models ; 27 pairs of optimal models at 3 sigma)

– **inferences without seismic data :**

* mass increases with $\alpha_{MLT}/ \alpha_{MLT,sun}$
* age can be found between 2 and 8 Gyr depending on the choice of the $\alpha_{MLT}/ \alpha_{MLT,sun}$ value

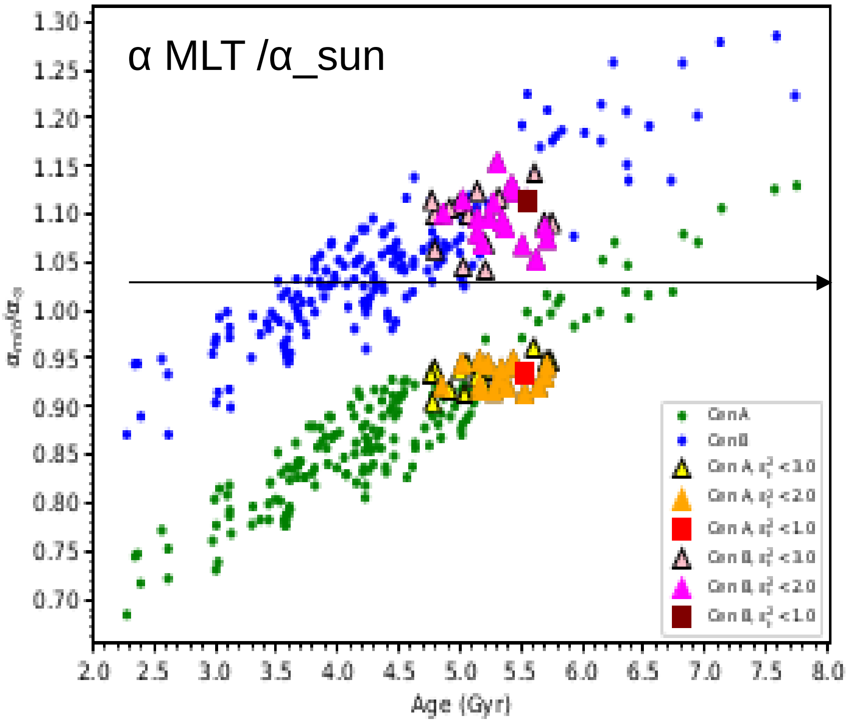
– **Inferences using seismic data**

* $\alpha_{MLT}/ \alpha_{MLT,sun}$ → same value independent of choices in input physics

* Correlation mass- α_MLT/α_MLT,sun :

$M_{Cen\,A}$ (1.10–1.11 M) > M_{sun} > M_{CenB} (0.93–0.94 M)
→
$\alpha_{MLT,A} /\alpha_{sun}$ = 0.6-0.8 < 1 < $\alpha_{MLT,B} /\alpha_{sun}$ = 0.8-0.11 1.095.

Joyce & chaboyer 2018



* Correlation age- α_MLT/α_MLT,sun
considerable drop in age scattering
2-8 Gyr → 4.8-5.7

in agreement with Yildiz+2017, ….

Impact of uncertainties on free parameters values : mixing length

- **Empirical determination** *(Bonacca 2012, Viani+ 2018, Creevey + 2017)*

Creevey + 2017 : inference for 57 stars of the legacy sample using AMP →

$$\alpha = 5.972778 + 0.636997 \log g - 1.799968 \log T_{eff} + 0.040094[M/H]$$

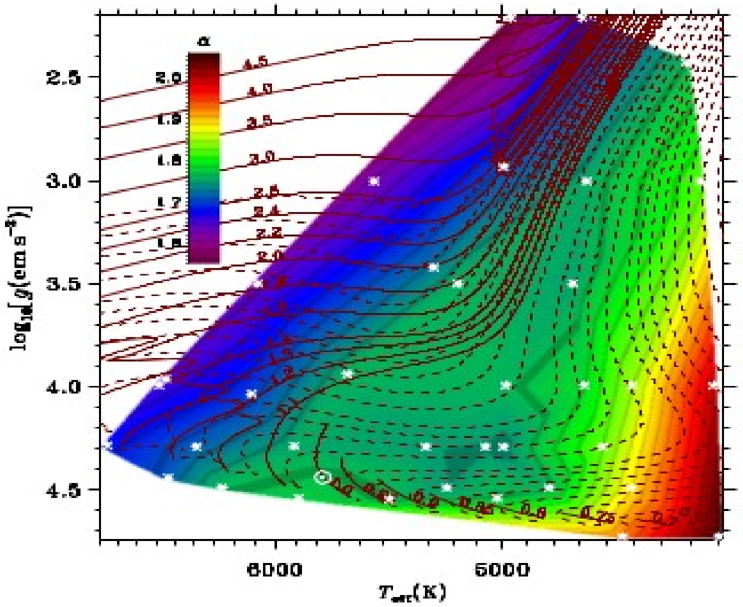
can serve as an inital guess for inference.
Joyce & Chaboyer 2018 results for alpha Cen AB are in agreement

- **Three-dimensional radiative hydrodynamic simulations of convection** predict that the mixing length should also depend on L, [Fe/H], log g

(*Ludwig+(1999), Trampedach+ (2014), Magic+ (2015))*

Joyce & Chaboyer (2018) results on AcenAB in disagreement with 3D simulations of convection (*Magic+ 2015*) for the sign of the metallicity dependence of our mixing-length

Trempedach + 2014



Mosumgaard et al 2017

Impact of priors on alpha and Yin

Free parameters are free to adjust but with some prior

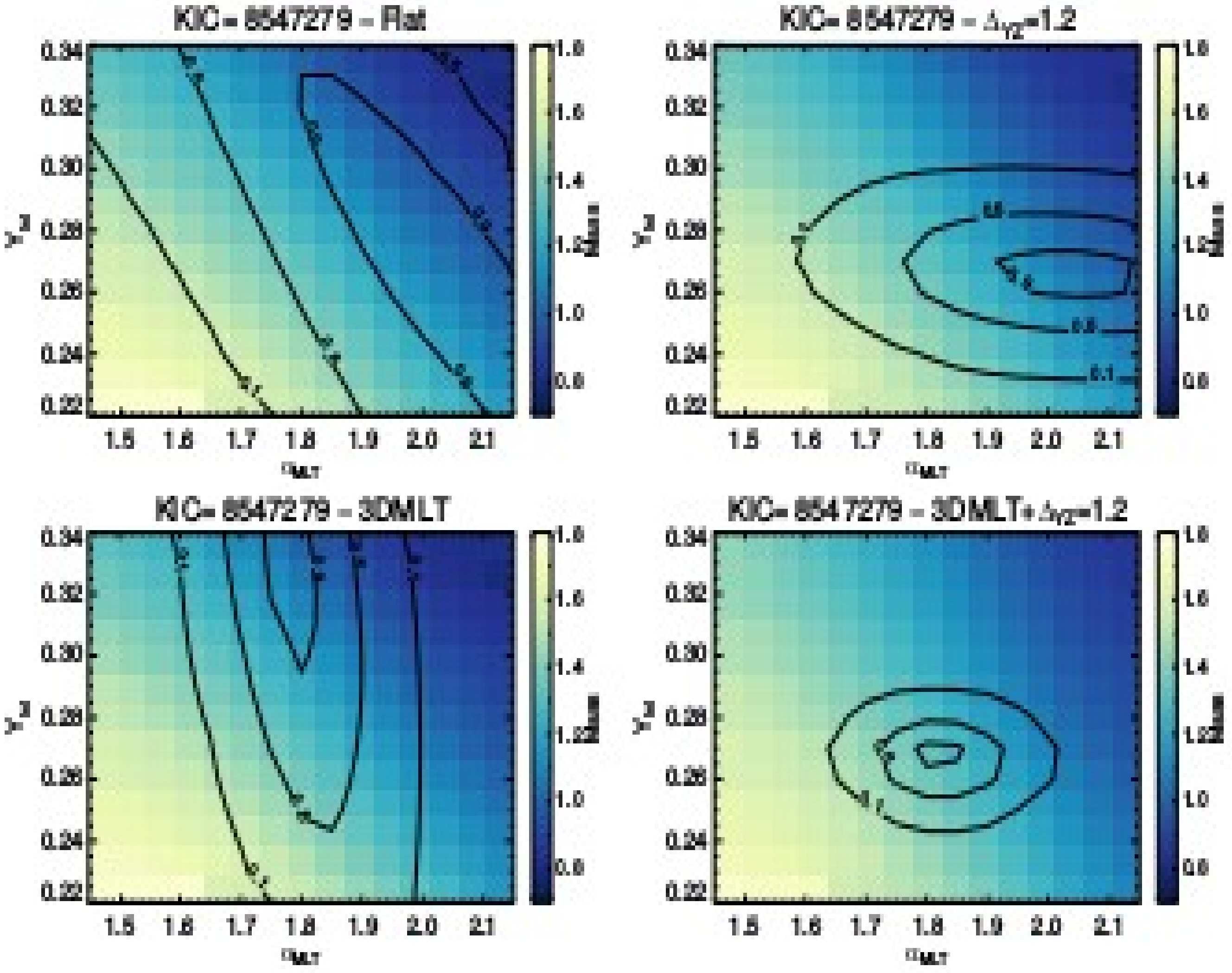
Prior 1 : Y_{ini}=Yp + 1.2 Z

σ = 0.01

Prior 2 : α_{MLT} = α_{3D} (Teff , log g, [Fe/H])

(from Magic+2015 but shifted by a constant value to match the solar calibrated value α_{MLT} = 1.802) and σ_{α} =0.05.

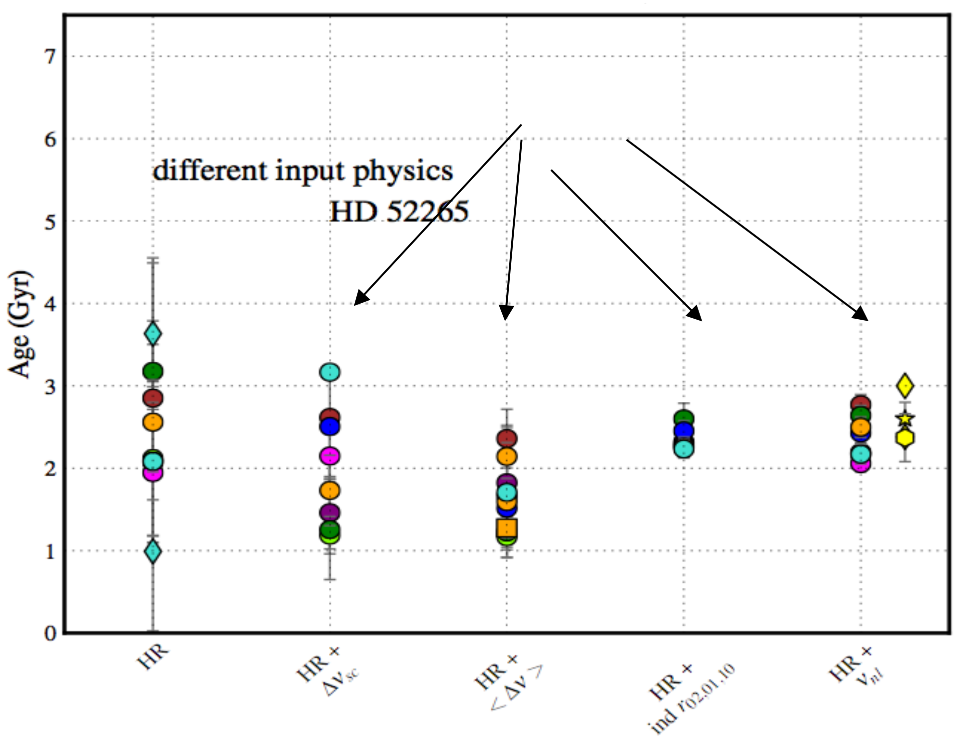
Uncertainty in mass ~ 4.4% deviation between the two most extreme cases: flat priors on Y_{ini} and α_{MLT} and priors on Yini and α



KIC 8547279 *Sereneli 2017*

Impact of inferring the free parameters

With seismology



Ex : **HD52265**

Escobar et al. (2012)

$1.24 \pm 0.02\ M_{\odot}$ $1.33 \pm 0.02\ R_{\odot}$, 2.6 ± 0.2 Gyr. (8%)

The error bars on these values do not include the impact of the uncertainties on stellar model inputs

- Lebreton (2013);Lebreton &Goupil (2014)

7 % on mass, 3 % radius, 13 % age

A = 2.10−2.54 Gyr, M/M_{sun} =1.14−1.32, R/R_{sun} = 1.30−1.34,

Δage/age= 13

âge 'classique'
0.8- 5.9 Gyr Δage/age= 75%

âge sismique
2.1-2.7 Gyr

Major sources of uncertainties in stellar modelling for (M,R,A) inferences :

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- initial helium content Yini
- solar chemical relative abundances (« mixture »)
- mixing length

2) The next most important sources of uncertainties depend on the mass of the star

Major sources of uncertainties in stellar modelling for (M,R,A) inferences :

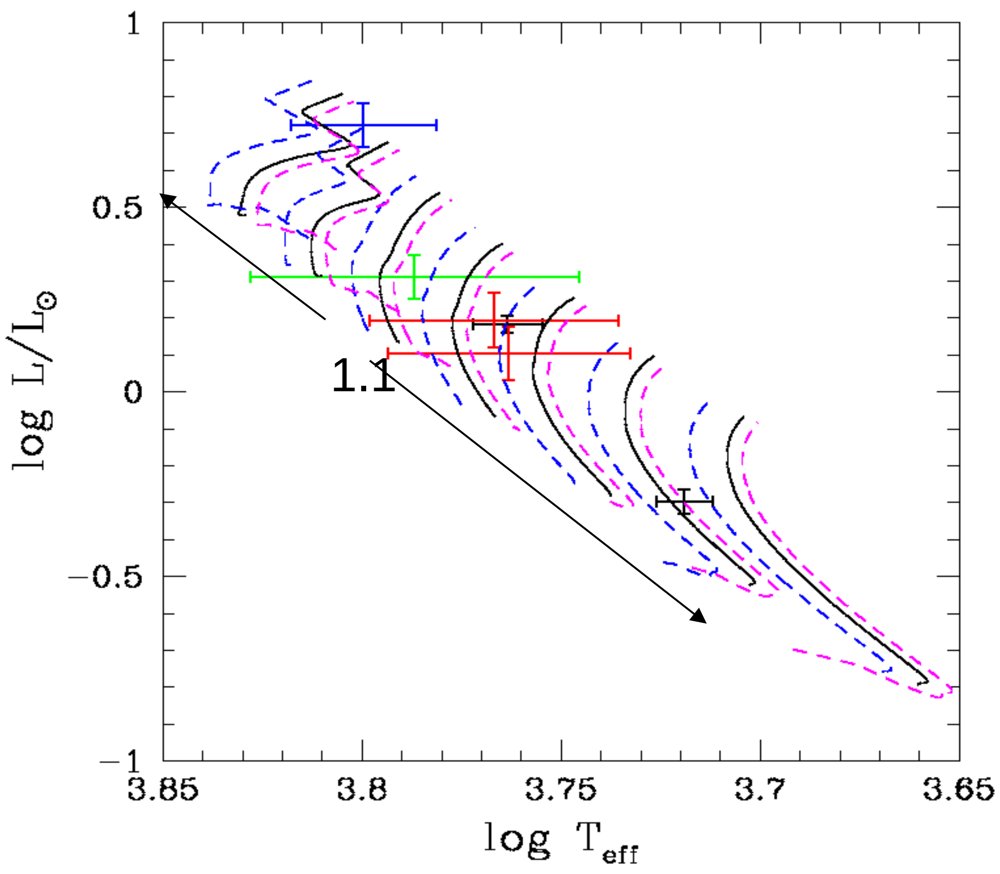
Convective core :M >~ 1.1-1.2 M_{sun}

- **α_{ov}**
- $^{14}N(p,gamma)^{15}O$ Pour M> ~ 1.3-1.4 M_{sun}
- radiative acceleration
- Rotation

No convective core, extended convective outer layers :

M <~ 1.1-1.2 M_{sun}

- **Atomic diffusion** : - inclusion or not
  - 20 % uncertainty on efficiiency
- Chemical mixture - assimilated to solar chemical mixture
- Opacities
- α_{MLT}/(MLT/CGM)/3D
- Nuclear (lowest side of age error bar)



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- $M/M_{sun} < 1.2$ Atomic diffusion

Impact of uncertainties on atomic diffusion

- Inclusion of atomic diffusion has a large impact on age

(Miglio & Montalban 2005   Joyce Chaboyer 2018 , Valle et al 2013, 2014, 2015 ,Silva Aguirre+2015)   *ΔM/M<~ 6 % ΔA/M<~ 20 %*

It reduces the age at turn-off of low-mass stars by a few per cent (*Lebreton & Goupil, 2014)*

- For **masses < 1.2 Msun** atomic diffusion matters while radiative acceleration are negligible

Still 20 % uncertainties on diffusion efficiency (Thoul 1994)

Ex : alpha Cen A,B : inferences favor models with standard diffusion η_D = 1.0 and   models with suppressed diffusion (η_D = 0.5) over models with enhanced diffusion (η_D = 1.5)

- For **masses > 1.2 Msun** , atomic diffusion drains the thin outer convective region of its heavy elements and helium

However radiative acceleration can hamper the drain of heavy elements . Remains the problem of helium depletion → usually thought to be counteracted by some turbulent mixing

16 CygA   *Buldgen 2016*

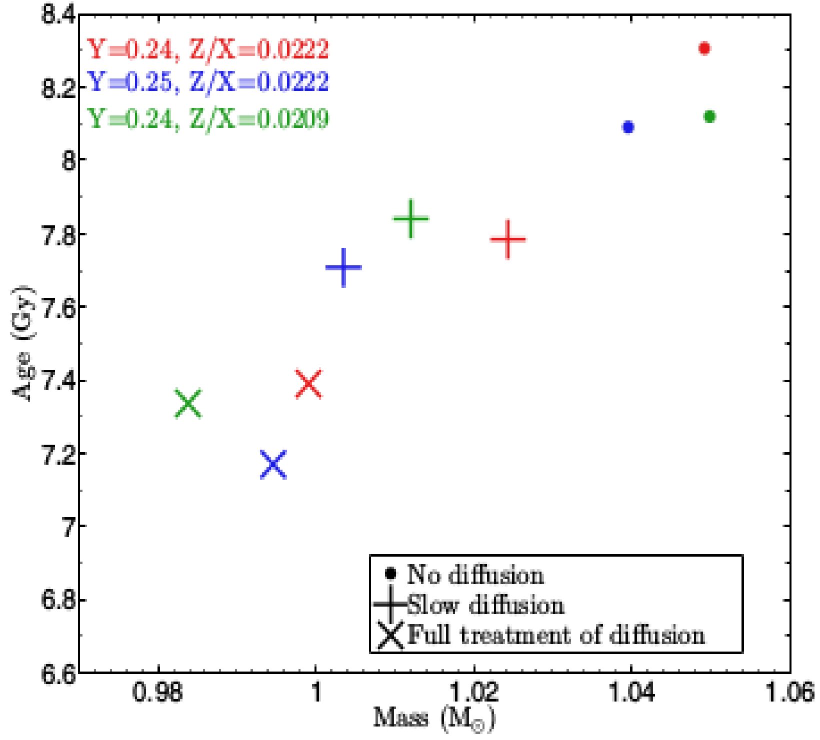


Fig. 2. Effect of the progressive inclusion of diffusion in a model of 16CygA. Each model still fits the observational constraints.

	No	diffusion/2	diffusion
Mass	1.052	1.025	1.002
Radius	1.240	1.229	1.218

In-depth study of uncertainties on grid-bases estimates of stellar mass and radius

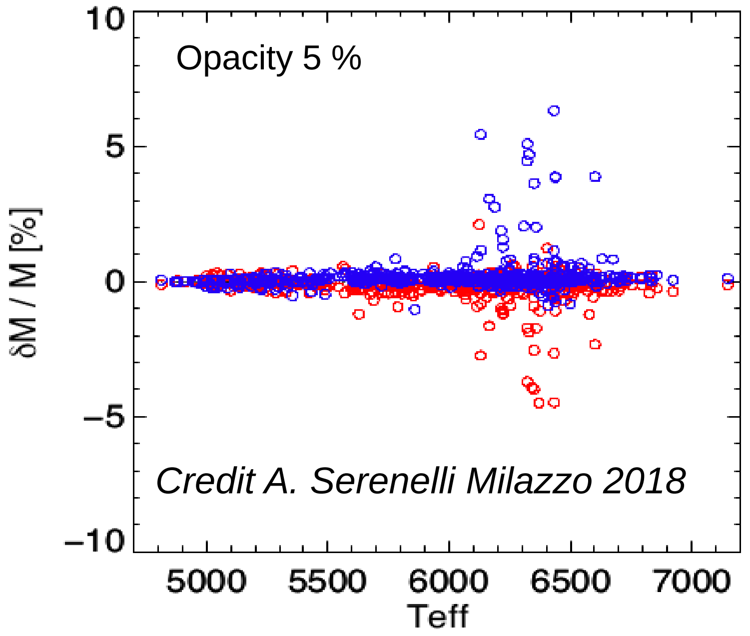
[0.8-1.1] Msun MS stars

Valle 2014 : grid-based inferences

- **Contraints :**Teff, [Fe/H], Delta nu , numax
  sigma : 100 K 0.1 2.5 % 5 %

- Impact of propagation of observational error

→ statistical error 4.5 %-2.2 % on mass and radius resp
(but up to 20 %, 10 % in some individual case)



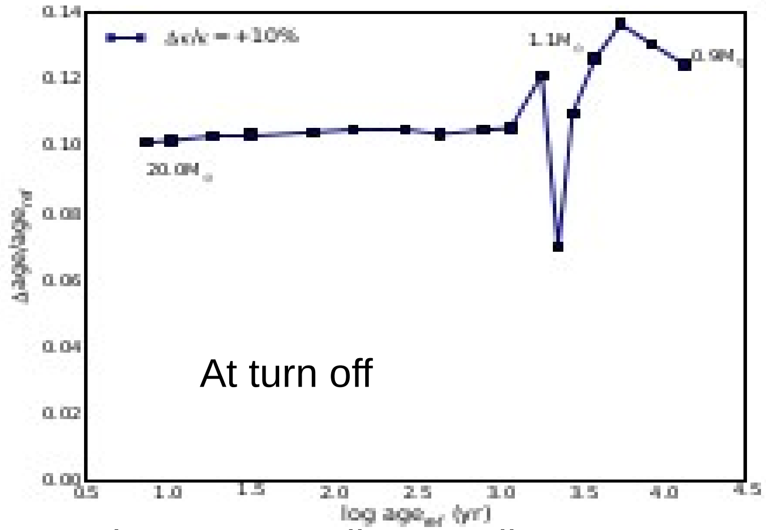
- Changes

		M/M_{sun}	R/R_{sun}
±1 % in ΔY/ΔZ	→	±2.3 %	±1.1 %
±0.24 in α_{MLT}	→	±2.1 %	±1.
±5 % opacity	→	± 1. %	±0.45 %
Neglect atomic diffusion	→	±3.7 %	±1.5 %

- Systematic errors smaller than statistical errors when varying one item at a time
- Varying two items at a time showed that **single one errors can be added .**

α_{MLT}+ ΔY_{ini} → **4.3 % on mass and 2.0 % on R**

→ same order of magntitude than statistical errors



Lebreton, Goupil, Montalban 2014

In-depth study of uncertainties on grid-bases estimates of stellar mass and radius

Kepler legacy data

- *Nsamba 2018* A study of 34 Kepler legacy stars

ΔY /ΔZ fixed to 2, no overshoot, no radiative levitation, α_{MLT} is inferred

	Density	R	M	A	
➢ Impact of diffusion :	0.5%	0.8%	2.1%	16%	syst in age >> statistical uncert.
➢ Solar mixture	0.7%	0.5%	1.4%	6.7%	
➢ Surface correction *(Sonoi+2015 : Ball & Gizon2014)*	~1%	~1%,	~2%,	~8%	

- *Silva Aguirre et al. , 2015 :* 33 Kepler planet-candidate host stars → systematics of ~1% in radius and density, ~2% in mass, and ~7% in age but $\alpha_{MLT} = \alpha_{MLT,sun}$

.

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- solar chemical relative abundances (« mixture »)
- mixing length

2) The next most important sources of uncertainties depend on the mass of the star

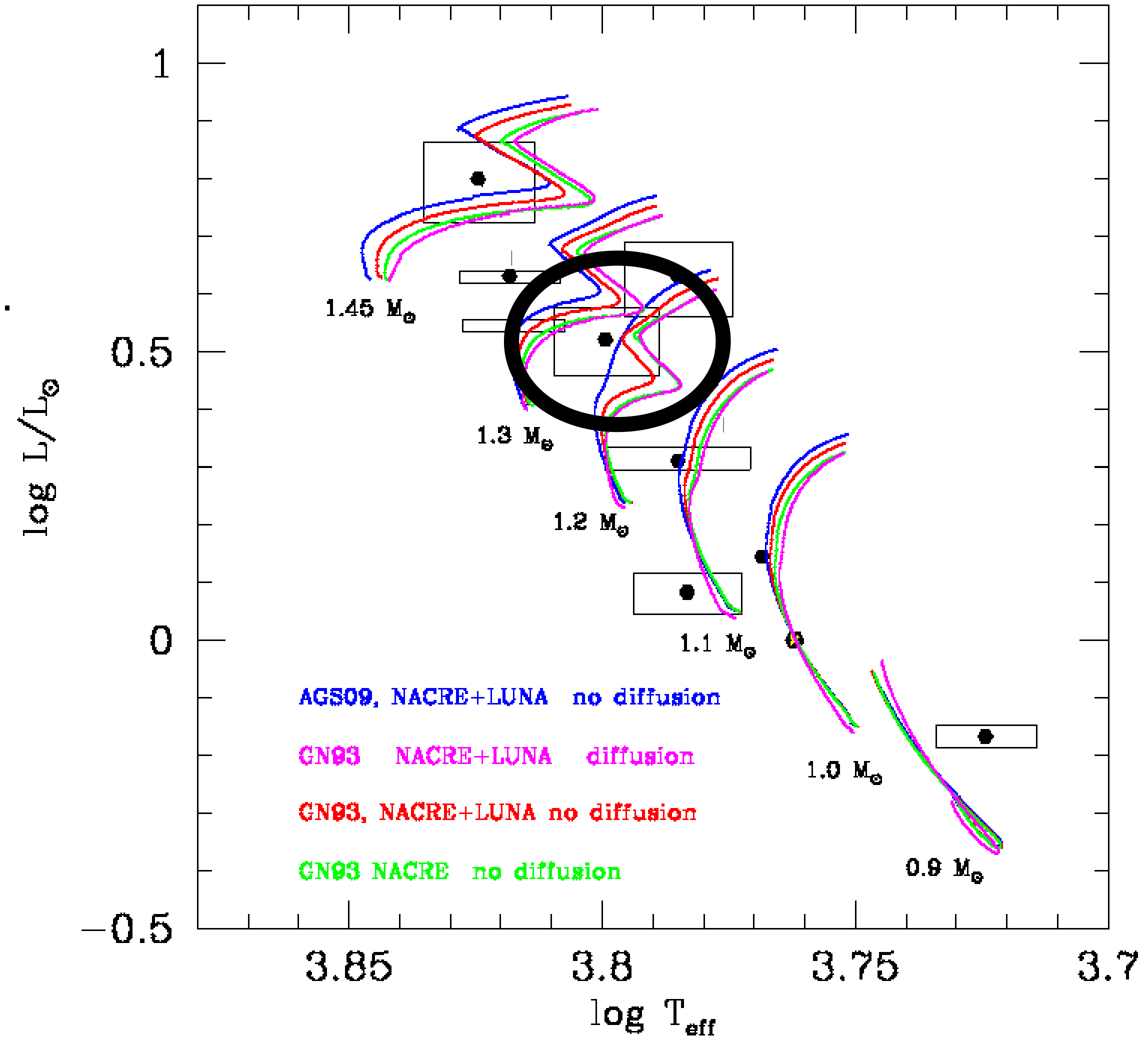
- $M/M_{sun} < 1.1- 1.2$ Atomic diffusion
- $M/M_{sun} > 1.1-1.2$ Core overshoot

Mass at onset of convective core

Onset of convective core

The mass at the onset of a convective core in stellar models depends on the solar mixture. The convective core appears higher mass for the lower metallicity given by the AGSS09. It depends also on efficiency of nuclear reaction of the CNO cycle.

CoRoT solar-like oscillators



Goupil+2010

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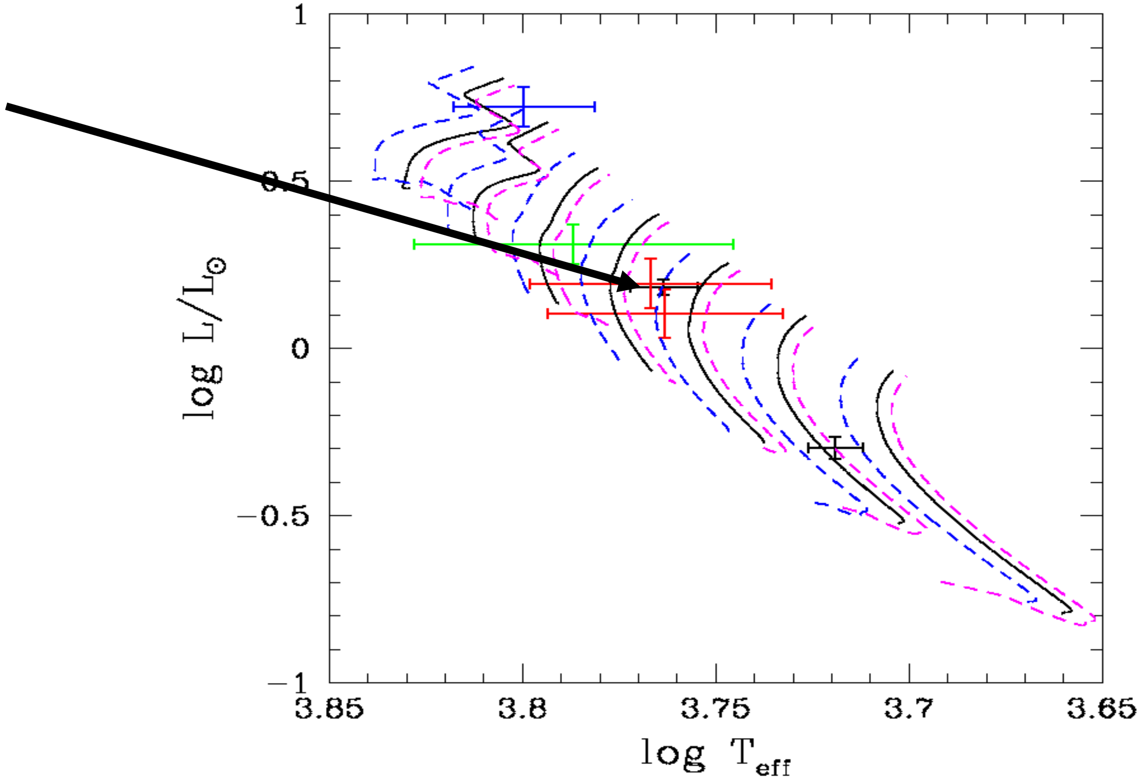
Ex :α Cen A :

- *Bazot+ (2016)* report a 40% chance of a convective core.
  Fit only Cen A (mixture GN93)

- Nsamba+ (2018) report a 70% chance of core convection.
  Fit only Cen A (mixture GS98)

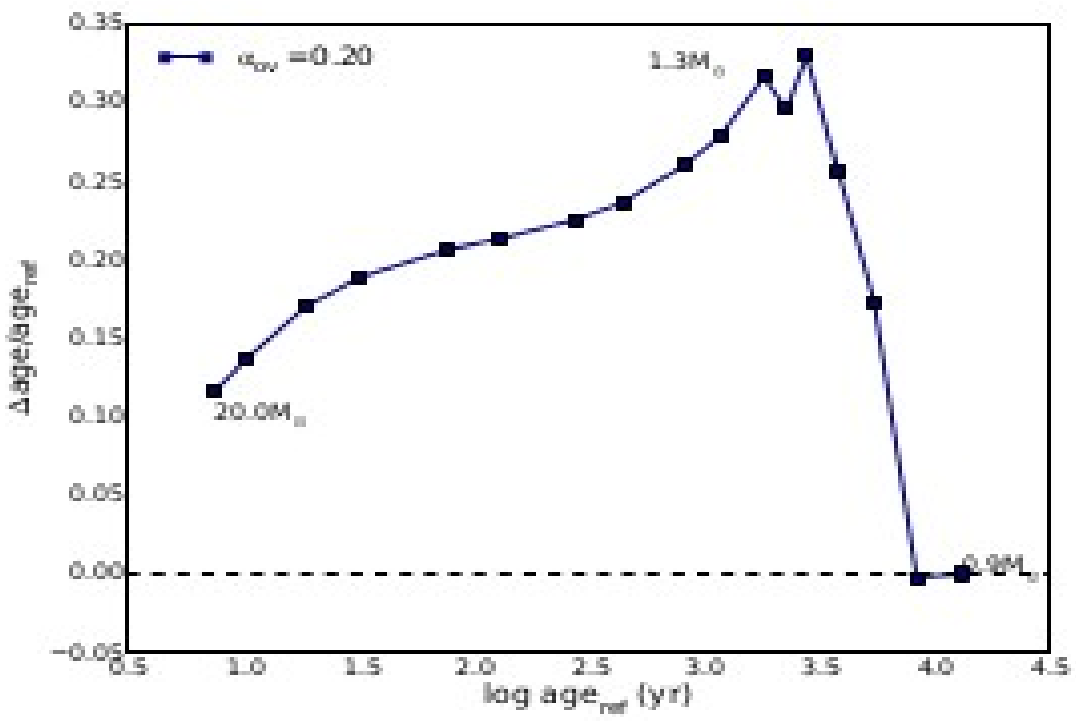
- *Joyce & Chaboyer 2018* fit both CenA,B
  A convective core can exist when diffusion in enhanced compared to standard (η_D = 1.5) but are not the most
- optimal models. Fitted ages lower by ~0.5 Gyr than models without. (mixture AGSS09)



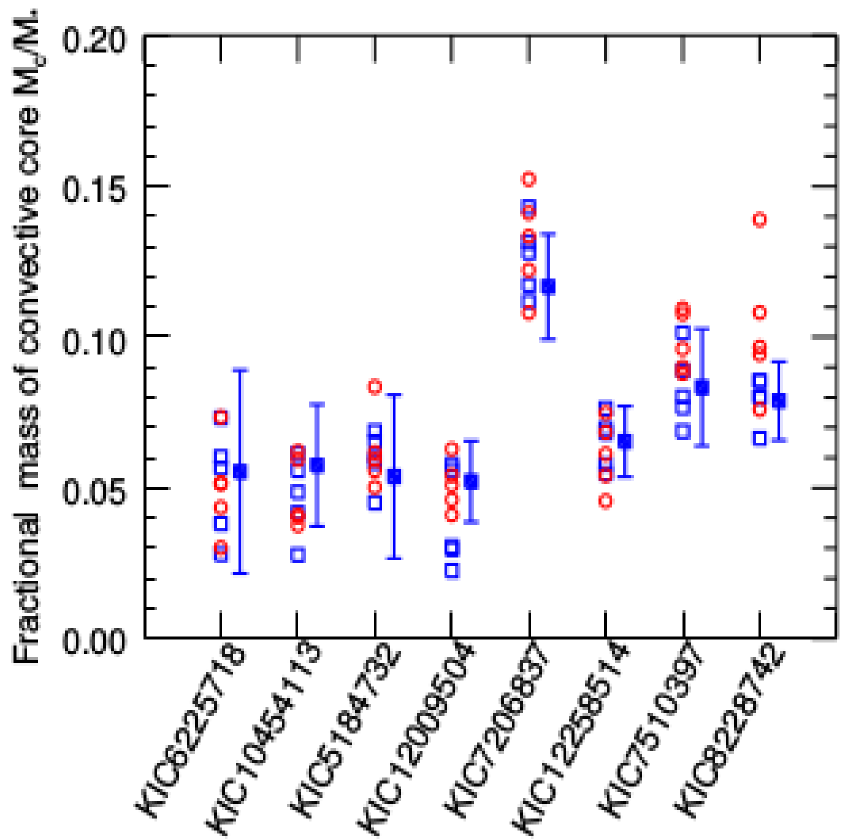
Impact of uncertainties on core overshoot

Changing the amount of convective overshoot can significantly change the recovered stellar properties ; the age at the turn off increases drastically with the onset of a convective core

Differences larger than 10 % for $M/M_{sun} > 1.1$



Lebreton & Goupil 2014

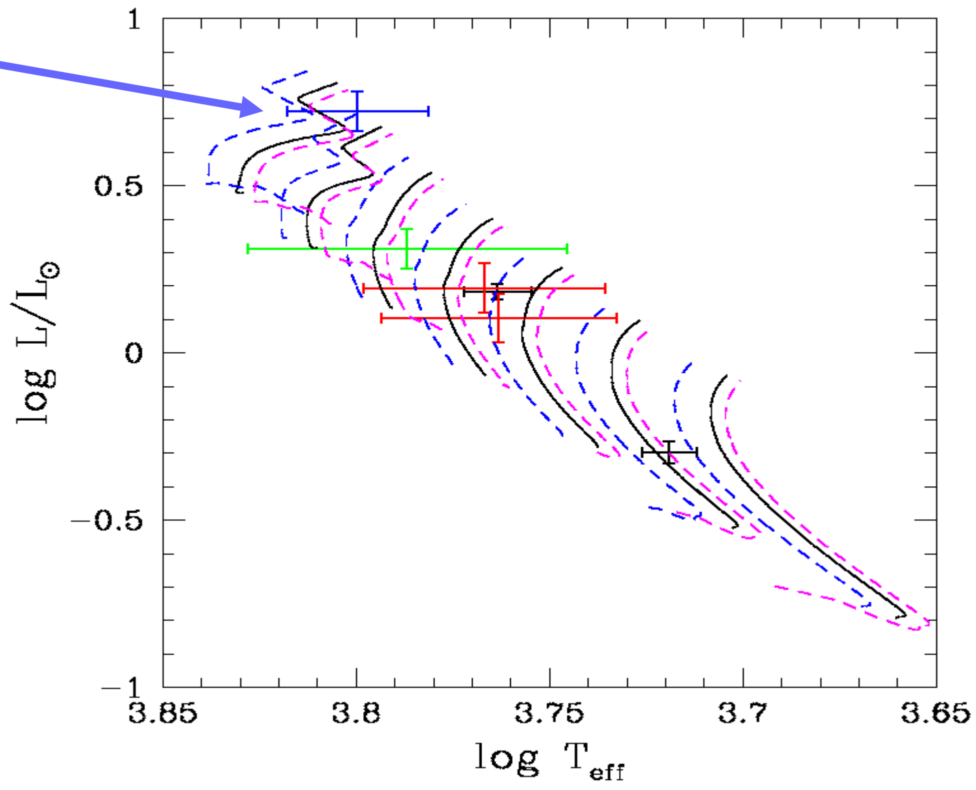


Deheuvels+2016

Impact of uncertainties on core overshoot

Ex: Kepler 21 a V = 8.25 F6IV star

Howell+2012 , Lopez Morales+2016



Observational constraints :
Teff , metal rich [Fe/H]=0.22 ± 0.05
Hipparccos distance → L_{obs}/L_{sun} = 5.54 ± 0.78
Seismic data *(Lund+2016)*

Ajusted parameters : Mass, age, L/L_{sun}

Free parameters : Y_{ini}, Z_{ini}, $(Z/X)_{in}$, α_{MLT}, α_{ov}

Inferences (*Silva Aguirre+ 2015*) show a bimodal mass distribution

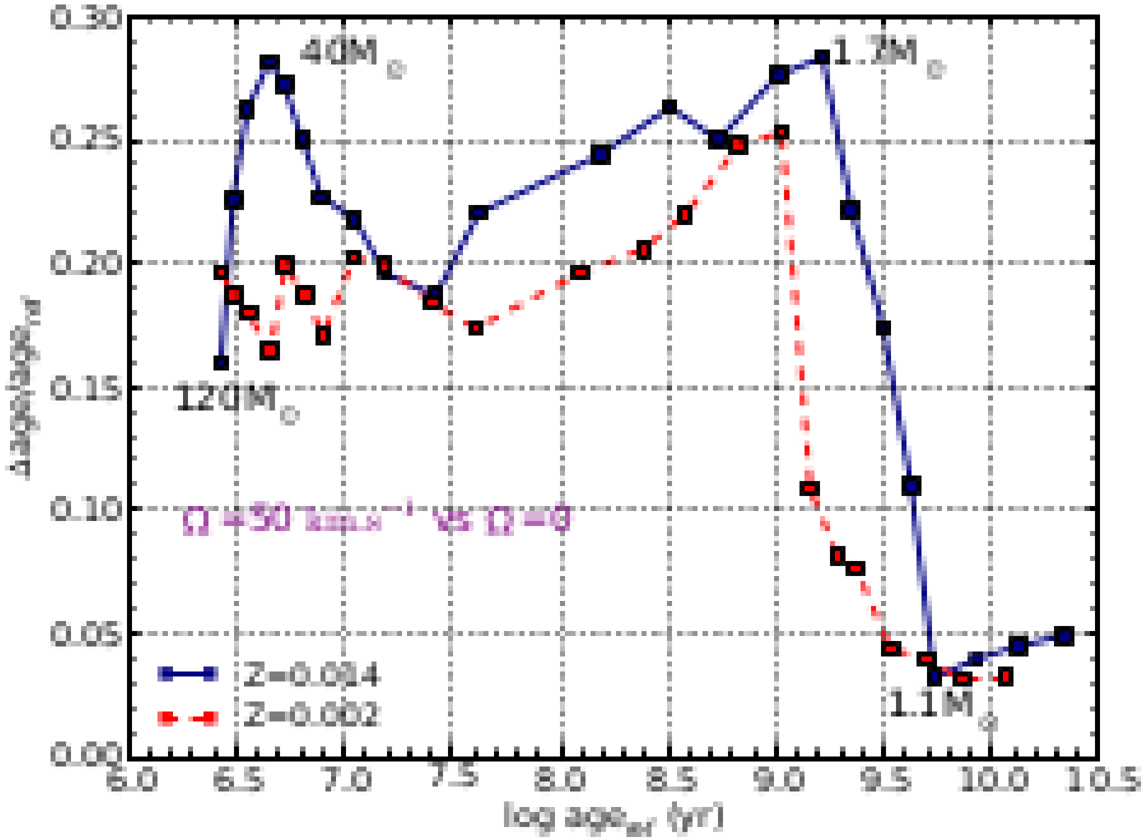
- BASTA used a grid including core overshooting which favoured the high mass value. L/L_{sun} agrees with Lobs/Lsun
  The BASTA low mass luminosity L/L_{sun} = 4.80 +0.12 −0.10
- ASTFIT → L/L_{sun} = 4.59 ± 0.11 < L_{obs}/L_{sun}

ASTFIT favoured the low-mass solution in the for Kepler-21

Rotationally induced transport

Transport of chemicals : rotationnaly induced mixing
Turbulent mixing

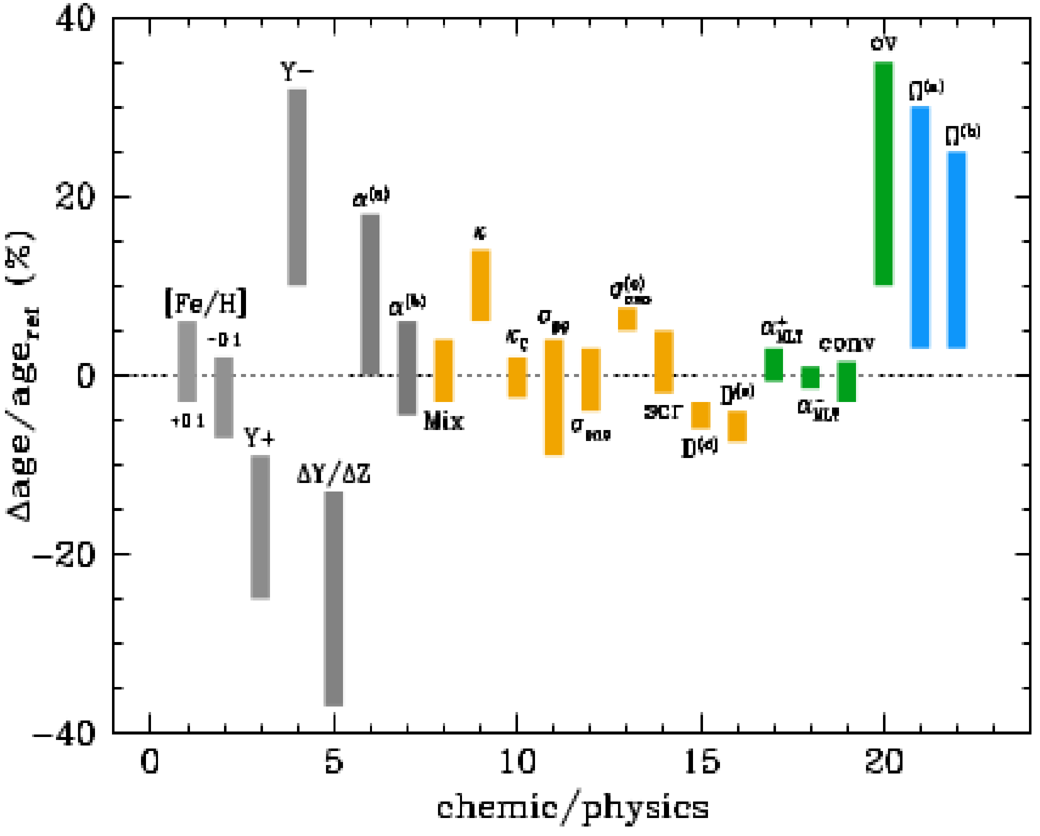
Differences up to 10 % for M/M_{sun}> 1.15 between Geneva models wtihand without rotationally induced mixing .



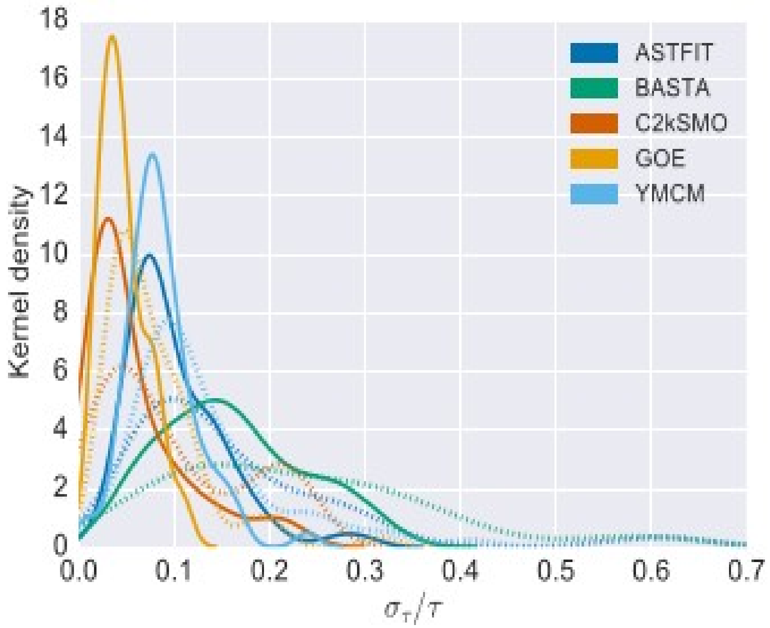
Lebreton, Goupil & Montalban 2014

Impact of uncertainties on mass, age in a nutshell

Uncertainties at turn off : relative differences compared to reference values



Lebreton, Goupil, Montalban, 2014



(Silva Aguirre+ 2017)

- Inferences of the Kepler legacy stars by several pipelines : me uncertainties
  0.5%- 2.6% in density;
  1.3%–4.2% in radius;
  2.3%– 4.5% in mass;
  6.7%–20% in age

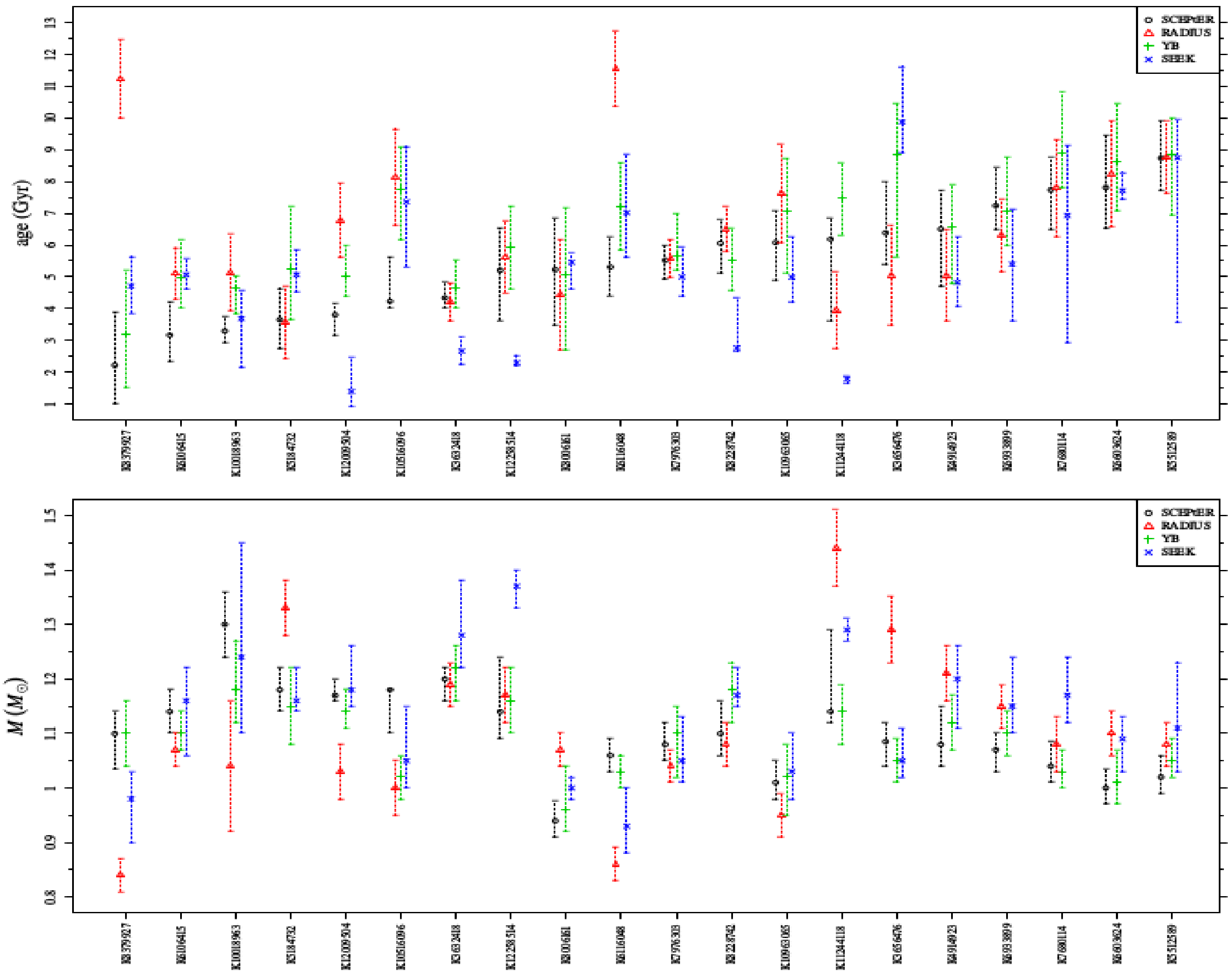
Results from different codes

Valle 2015

Comparison in estimates of mass and radius for 7 stars from different codes often larger than statistical errors :

The results between codes significantly differ for many stars → can have significant impact on planet characterisation

On the other hand, agreement for the benchmark stars Alpha Cen and Theta Ophiucis Additional constraints: mass from binarity, radius from interferometry

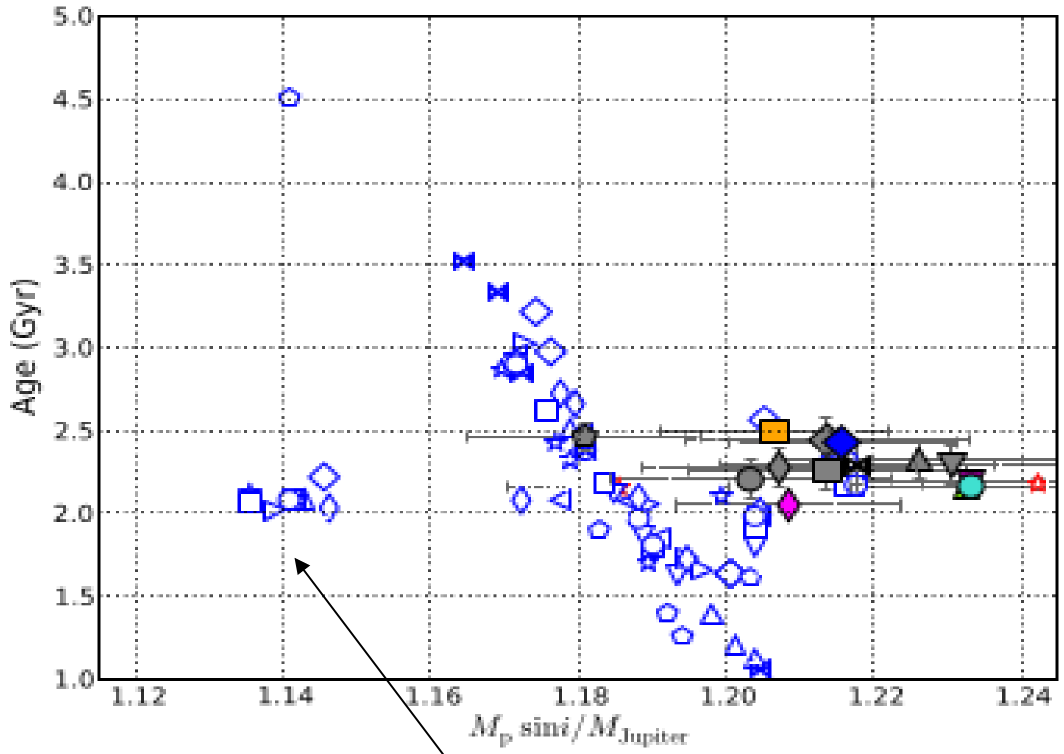


ig. 13. SCEPtER age and mass estimates for the observational sample from Mathur et al. (2012), compared with those by RADIUS, YB, and EEK. Objects were sorted by ascending SCEPtER estimated age.

Impact on planet characterisation

HD52265

- Mp,min = 1.13 ± 0.03 $M_{Jupiter}$ *(Butler 2000)* no age
- Mp,min = 1.09 ± 0.11 $M_{Jupiter}$ (i.e. sin i = 1) *(Gizon+ 2013)*
- Mp sin i = 1.16−1.26 $M_{Jupiter}$ *(Lebreton & Goupil 2014)*



and many other exemples… Kep21 for one ….

Thegroup of models with low mass and age of ~2 Gyr are case 4 models, to be rejected because of their high initial helium abundance

Some conclusions

1) Major uncertainties in stellar modelling are due to the existence of free parameters. They are pf different nature :

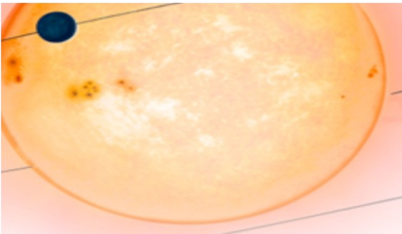
- those due to improper modelling of a physical process , mainly macroscopic processes
Theoretical work ought to remove or at least alleviate the problem and are in progress
2D (rotation)/ 3D modelling(convection)

- those which are intrinsic to individual stars : initial helium abundance, initial heavier elements abundances
Measurements should provide these quantities at least for the brightest stars with higher quality data

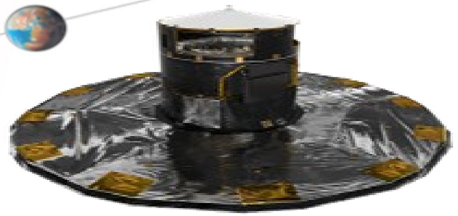
2) Differences in the results of different codes must be identified before using the mass, radius and age values of the star for star/planet studies

3) Physics description is continuously improving : microphysics (lab measurement (EOS), theoretical work (opacities)), macrophysics (transport processes), surface convection and patched models, …

All this needs high quality data ….



Perspectives observationnelles

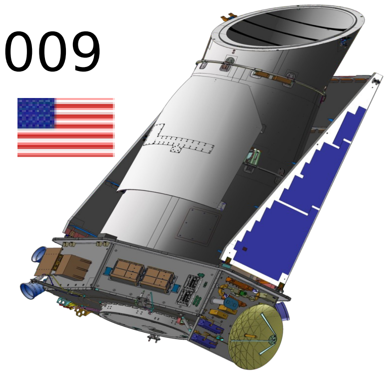


GAIA

CoRoT

2007

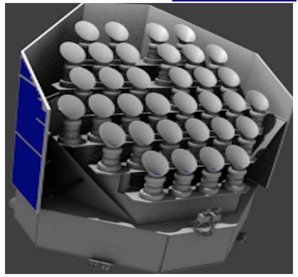
2009



2018

TESS

2026



PLATO

~10 *

5 months

Kepler/K2

500 * detection
99 *   analysed

4 years

27d-1year

2-4 years +extension